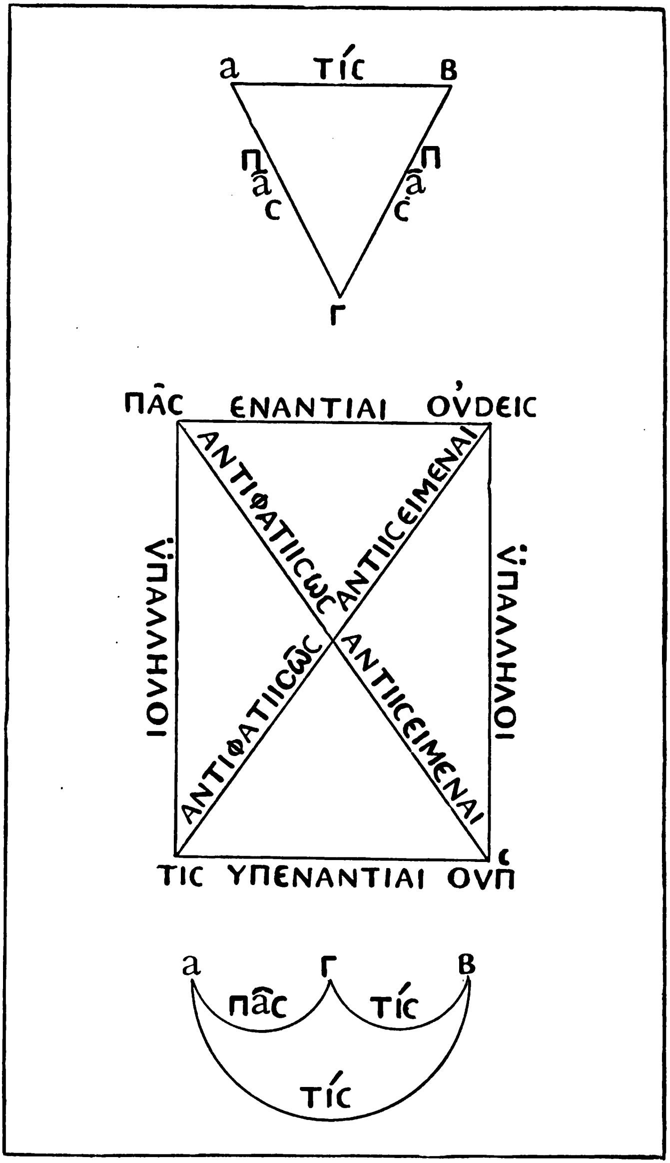
STUDIES IN DEDUCTIVE LOGIC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STUDIES

IN

DEDUCTIVE LOGIC

A Manual for Students

BY

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LL.D. (EDINB.), M.A. (LOND.), F.R.S.

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PREFACE

In preparing these 'Studies' I have tried to carry forward the chief purpose of my *Elementary Lessons in Logic*, which purpose was the promotion of practical training in Logic. In the preface to those Lessons I said in 1870: 'The relations of propositions and the forms of argument present as precise a subject of instruction and as vigorous an exercise of thought, as the properties of geometrical figures or the rules of Algebra. Yet every schoolboy is made to learn mathematical problems which he will never employ in after life, and is left in total ignorance of those simple principles and forms of reasoning which will enter into the thoughts of every hour. . . . In my own classes I have constantly found that the working and solution of logical questions, the examination of arguments and the detection of fallacies, is a not less practicable and useful exercise of mind than is the performance of calculations and the solution of problems in a mathematical class.'

The considerable use which has been made of the *Elementary Lessons* seems to show that they meet an educational want of the present day. The time has now perhaps arrived when facilities for a more thorough course of logical training may be offered to teachers and students.

For a long time back there have been published books containing abundance of mathematical exercises, and not a few works consist exclusively of such exercises. In recent years the teachers of other branches of science, such as Chemistry and the Theory of Heat, have been furnished with similar collections of problems and numerical examples. There can be no doubt about the value of such exercises when they can be had. The great point in education is to throw the mind of the learner into an active, instead of a passive state. It is of no use to listen to a lecture or to read a lesson unless the mind appropriates and digests the ideas and principles put before it. The working of problems and the answering of definite questions is the best, if not almost the only, means of ensuring this active exercise of thought. It is possible that at Cambridge mathematical gymnastics have been pushed to an extreme, the study of the principles and philosophy of Mathematics being almost forgotten in the race to solve the greatest possible number of the most difficult problems in the shortest possible time. But there can be no manner of doubt that from the simple addition sums of the schoolboy up to problems in the Calculus of Variations and the Theory of Probability, the real study of Mathematics must consist in the student cracking his own nuts, and gaining for himself the kernel of understanding.

So it must be in Logic. Students of Logic must have logical nuts to crack. Opinions may differ, indeed, as to the value of logical training in any form. That value is twofold, arising both from the general training of the mental powers and from the command of reasoning processes eventually acquired. I maintain that in both ways Logic, when properly taught, need not fear comparison with the Mathematics, and in the second point of view Logic is decidedly superior to the sciences of quantity. Many students acquire a wonderful facility in integrating differential equations, and cracking other hard mathematical nuts, who will never need to solve an equation again, after they settle down in the conveyancer's chambers or the vicar's parsonage. With the ordinary forms of logical inference and of logical combination they will ceaselessly deal for the rest of their lives; yet for the knowledge of the forms and principles of reasoning they generally trust to the light of nature.

I do not deny that a mind of first-rate ability has considerable command of natural logic, which is often greatly improved by a severe course of mathematical study. But I have had abundant opportunities, both as a teacher and an examiner, of estimating the logical facility of minds of various training and capacity, and I have often been astonished at the way in which even well-trained students break down before a simple logical problem. A man who is very ready at integration begins to hesitate and flounder when he is asked such a simple question as the following: 'If all triangles are plane figures, what information, if any, does this proposition give us concerning things which are not triangles?' As to untrained thinkers, they seldom discriminate between the most widely distinct assertions. De Morgan has remarked in more than one place[1] that a beginner, when asked what follows from 'Every A is B,' answers 'Every B is A *of course*.' The fact that such a converse is often true in geometry, although it cannot be inferred by pure logic, tends to mystify the student. Although all mathematical reasoning must necessarily be logical if it be correct, yet the conditions of quantitative reasoning are often such as actually to mislead the reasoner who confuses them with the conditions of argumentation in ordinary life. A mathematical education requires, in short, to be corrected and completed, if indeed it should not be preceded, by a logical education. There was never a greater teacher of mathematics than De Morgan; but from his earliest essay on the Study of Mathematics to his very latest writings, he always insisted upon the need of logical as well as purely mathematical training. This was the purpose of his tract of 1839, entitled, *First Notions of Logic preparatory to the Study of Geometry*, subsequently reprinted as the first chapter of the *Formal Logic*. A like idea inspired his valuable essays *On the Method of Teaching Geometry*, quoted above.

[1] *The Schoolmaster:* Essays on Practical Education, 1836, vol. ii. p. 120, note. This excellent essay 'On the Method of Teaching Geometry' was originally printed in the *Quarterly Journal of Education*, No. XI. 1833, vol. vi. pp. 237-251. Similar views are put forth in De Morgan's earlier work, *On the Study and Difficulties of Mathematics*, published in 1831 by the Society for the Diffusion of Useful Knowledge. See chapter xiv. See also De Morgan's Fourth Memoir on the Syllogism, p. 4, in the *Cambridge Philosophical Transactions* for 1860.

Professor Sylvester, indeed, in his most curious tractate upon the *Laws of Verse* (p. 19), has called in question the nut-bearing powers of logic, saying: 'It seems to me absurd to suppose that there exists in the science of pure logic anything that bears a resemblance to the infinitely developable and interminable euristic processes of mathematical science.' To such a remark this volume is perhaps the best possible answer, especially when it is stated that I have had great difficulty in selecting and compressing my materials so as to get them into a volume of moderate size. If any person who thinks with Professor Sylvester should object to the greater part of the problems as dealing with concrete logic, let him look to the end of this book, where he will find that the closely printed Logical Index to the forms of law governing the combinations of only three terms, fills four pages, without in any way including the almost infinitely various logical equivalents of those distinct forms. He will also learn that a similarly complete index of the forms of logical law governing the combinations of only five logical terms would fill a library of 65,536 volumes. Surely there is scope enough here for 'euristic processes.'

An anxious and difficult task which I had to encounter in compiling this book consisted in choosing the system or systems of logical notation and method which were to be expounded. When once the convenient but tyrannical uniformity of the Aristotelian logic was overthrown, each writer on the science proceeded to invent a new set of symbols. But it is impossible to employ alike the Greek letters of Archbishop Thomson, the 'mysterious spiculae' of De Morgan, the cumbrous strokes, wedges, and dots of Sir W. Hamilton, and the intricate mathematical formulae of Boole. After a careful renewed study of the writings of these eminent logicians I felt compelled in the first place to discard the diverse and complicated notative methods of De Morgan. Few or none admire more than I do the extraordinary ingenuity, fertility, and, in a certain way, the accuracy of De Morgan's logical writings. My general indebtedness, both to those writings and to his own unrivalled oral teaching, cannot be sufficiently acknowledged. I have, moreover, drawn many particular hints from his works too numerous to be specified. Nevertheless, to import his 'mysterious spiculae' into this book was to add a needless stumbling-block. The question would have arisen too, which of his various systems to adopt; for De Morgan created six equally important concurrent syllogistic systems, the initial letters of the names of which he characteristically threw into the anagrams, 'Rue not!' 'True? No!' These systems were the Relative, Undecided, Exemplar, Numerical, Onymatic, and Transposed. See *A Budget of Paradoxes*, pp. 202-3. There was in fact an unfortunate want of power of generalisation in De Morgan; his mind could dissect logical questions into their very atoms, but he could not put the particles of thought together again into a real system. As his great antagonist, Sir W. Hamilton, remarked, De Morgan was wanting in 'Architectonic Power.'

It seems equally impossible, however, to adopt Sir W. Hamilton's own logical symbols. His chief method of notation has been briefly described in the *Elementary Lessons in Logic* (p. 189). He also constructed or contemplated other systems of notation, as stated in his *Lectures on Logic* (vol. iv. pp. 464-476). In no case do these notations seem to be so good as the earlier and simpler one of Mr. George Bentham. And after a laborious reinvestigation, rendered indispensable by the composition of various parts of this book, I have been forced to the conviction that in almost every case where Hamilton differed from contemporaries or predecessors he blundered. He was, as his admirers said, to put the keystone into the arch of the Aristotelic syllogism; but, in spite of his 'Architectonic Power' I fear we must allow that *his* arch has collapsed. (See pp. 129-133, 151-4, and 157-8, of this book.)

With the logical innovations of Dr. Thomson the case is different. While he appears to enjoy the credit of an independent discovery of the Quantification of the Predicate, prior to any public and explicit statement of the same by Hamilton, De Morgan, or Boole, but posterior to the neglected work of Mr. George Bentham, he did not commit the blunders of Hamilton, nor overlay his work with useless crowds of shorthand symbols. He most aptly completed the ancient scholastic notation of propositions (**A, E, I, O**) by adding **U, Y,** η and ω to denote the new forms derived from Quantification of the Predicate, carefully showing at the same time that η and

ω are practical nonentities. I have therefore used his notation for quantified propositions and syllogisms where necessary.

Boole's great works are of course the foundation of almost all subsequent progress in formal logic. My own views, as I long since explicitly stated,[1] are moulded out of his. Believing, however, that the mathematical dress into which he threw his discoveries is not proper to them, and that his quasi-mathematical processes are vastly more complicated than they need have been, I have of course preferred my simpler version. Students who wish to comprehend Boole's power and Boole's methods must go to the original writings. It is really impossible that any abstract or summary can give an adequate idea of the stupendous efforts which Boole made to construct a general mathematical calculus of inference. Dr. Macfarlane, of Edinburgh, has lately published a new version of Boole's system under the title *Algebra of Logic*, but I am unable as yet to discover that he has made any improvement on Boole.

The writings of M. Delboeuf on Algorithmic Logic, first printed in the *Revue Philosophique* for 1876, and since reprinted, are very interesting, but were written in ignorance of what had been done in this country by Boole and others.

Quite recently Mr. Hugh MacColl, B.A., has published in the *Proceedings of the London Mathematical Society*, and in *Mind*, several papers upon a Calculus of Equivalent

[1] *Pure Logic*, 1864, p. 3, etc.

Statements, which arose out of an earlier article in the *Educational Times.*[1] His Calculus differs in several points both from that of Boole and from that described in this book as Equational Logic. Mr. MacColl rejects equations in favour of *implications;* thus my A = AB becomes with him A : B, or A implies B. Even his letter-terms differ in meaning from mine, since his letters denote propositions, not things. Thus A : B asserts that the statement A implies the statement B, or that whenever A is true, B is also true. It is difficult to believe that there is any advantage in these innovations; certainly, in preferring implications to equations, Mr. MacColl ignores the necessity of the equation for the application of the Principle of Substitution. His proposals seem to me to tend towards throwing Formal Logic back into its ante-Boolian confusion.

In one point, no doubt, his notation is very elegant, namely, in using an accent as a sign of negation. A′ is the negative of A; and as this accent can be applied with the aid of brackets to terms of any degree of complexity, there may sometimes be convenience in using it. Thus $(A + B)' = A'B'$; $(ABCD \ldots)' = A' + B' + C' + D' + \ldots$. I shall occasionally take the liberty of using the accent in this way (see p. 199), but it is not often needed. In the case of single negative terms, I find experimentally that De Morgan's Italic negatives are the best. The Italic *a* is not only far more clearly distinguished from A than is A′, but it is written with one pen-stroke less,

[1] August 1871, also July 1877.

which in the long run is a matter of importance. The student, of course, can use A′ for *a* whenever he finds it convenient.

The logical investigations of Mr. A. J. Ellis, F.R.S., require notice, because they are closely analogous to, if not nearly identical with, my own. I am much indebted to him for assisting me to become acquainted with his views. Not only has he supplied me with an unpublished reprint, with additions, of his articles in the *Educational Times*, but he has allowed me access to the manuscripts of two elaborate memoirs which he presented to the Royal Society, and which are now preserved in the archives of the Society. Some account of these investigations will be found in the *Proceedings of the Royal Society* for April 1872, No. 134, vol. xx. p. 307, and November 1873, vol. xxi. p. 497. In the former place Mr. Ellis remarks: 'The above contributions are believed to be entirely original . . . . Jevons first led my thoughts in this direction, but all resemblance between us is entirely superficial.' The question of resemblance thus raised by Mr. Ellis must be left to others to decide; but in order to avoid possible misapprehension, I must say, that however different in symbolic expression, Mr. Ellis's logical system seems to be identical in principle with my own. The developments of the Combinational Method, as described in the *Educational Times* (June, July, and August, 1872), are substantially the same as I had previously published in several papers and books Mr. Ellis also employs card diagrams of combinations arranged upon the ledges of a black-board, which practically form the Logical Abacus, as described by me in 1869.

The only point in which I am conscious of having received assistance from Mr. Ellis has regard to the necessary presence of combinations and the significance of their total disappearance as proving contradiction. I may not have sufficiently insisted upon the importance of this matter; but the fact is that so long ago as 1864 (see pp. 181, 192, of this book) I pointed out the complete disappearance of a letter-term from the combinations as the criterion of contradiction in the conditions governing logical combinations, and the same principle is explicitly stated in the *Principles of Science* (1874, vol. i. p. 133; new edition, p. 116). In the latter part of this book I have more fully developed the theory of the relation of propositions, often turning as it does upon this criterion of contradiction. This theory will, I think, be found to be the natural development of ideas stated in my earlier essays; but I may have received some hints from Mr. Ellis's writings. The above remarks apply only to such portions of Mr. Ellis's Memoirs as treat of logical combination and inference; other portions in which he investigates sequence in space and time, probability, etc., are not at all in question.

The Logical Index, although now printed for the first time, has been in my possession since 1871 (see *Principles of Science*, 1st edition, vol. i. pp. 157, 162; new edition, pp. 137, 141, etc.); but it is only by degrees that I have appreciated the wonderful power which it gives over all logical questions involving three terms only; and it is quite recently that it has occurred to me how it might be printed in the form of a compact and convenient table.

Mr. Venn has published in the *Philosophical Magazine* for July 1880, a paper 'On the Diagrammatic and Mechanical Representation of Propositions and Reasonings.' An article on 'Symbolic Reasoning' by the same author will also be found in *Mind* for the same month. The text of this book having been completed and placed in the printer's hands before Mr. Venn's ingenious papers were published, it has not been possible to illustrate or to criticise his views.

I may mention that M. Louis Liard, Professor of Philosophy at Bordeaux, who had previously explained and criticised the substitutional view of Logic in the *Revue Philosophique* (Mars, 1877, tom. iii., p. 277, etc.), has since published a very good though concise account of the principal recent logical writings in England, under the title, *Les Logiciens anglais contemporains* (Paris: Germer Baillière, 1878).

These 'Studies' consist in great part of logical Questions and Problems gathered from many quarters. In the majority of cases I have indicated by initial letters the source or authorship of the questions when clearly known (see the List of References on p. xxv); but I have not always carried out this rule, and in not a few cases the questions have been printed several times already, and are of doubtful authorship. A large remaining fraction of the questions and problems are new, and have been devised specially for this book. As shown by the author's name appended, a few questions have been borrowed from the work of the Very Rev. Daniel Bagot, Dean of Dromore, entitled *Explanatory Notes on the Principal Chapters of Murray's Logic . . . with an Appendix of* 337 *Questions to Correspond.* A few excellent illustrations have also been drawn from a privately printed tract on Logic by the late Sir J. H. Scourfield, M.P., his own annotated copy having been kindly presented to me by the author a few years before his death.

In forming this compilation I have been more than ever struck by the fact that the larger part of logical difficulties and sophisms do not turn upon questions of formal logic but upon the relations which certain assertions bear to the presumed or actual knowledge of the assertor and the hearer. If the person *X* remarks that 'All lawyers are honourable men,' it is one question what is the pure logical force of this proposition, as measured by its effect on the combinations of the terms concerned and their negatives. It is quite another matter what *X* means by it; why he asserts it; what he expects *Y* to understand by it; and what *Y* actually does take as the meaning of *X*.

Under certain circumstances assertions convey a meaning the direct opposite of what they convey at other times. If a man is taken with a fit and the first medical man who arrives says, 'You must not think of putting the man under the pump,' the man will not be put under the pump; but if the identically same assertion is made about the centre of interest of an excited and angry mob, the man goes to the pump. It is evident that there ought to exist a science of applied deductive logic, partly corresponding to the ancient doctrines of rhetoric, in which the popular force of arguments as distinguished from their purely logical force should be carefully analysed. A few questions and answers given in this book may perhaps belong, properly speaking, to rhetorical logic (see pp. 119, 140–1, etc.), but I have not found it practicable to pursue the subject in this book. It should be evident that a thorough comprehension of the purely logical aspect of assertions must precede any successful attempt to investigate their rhetorical aspect. I may possibly at some future time attack the problems of rhetorical logic.

A further question which forced itself upon my notice was that of the practicability of including exercises in Inductive Logic. As Mr. H. S. Foxwell suggested, inductive exercises and problems are even more needed than those of a deductive character. But, on consideration and trial, it seemed highly doubtful whether it would be possible to throw questions of inductive logic into the concise and definite form essential to a book of exercises. I have given abundance of inverse combinational problems which are really of an inductive character (see pp. 252–8); but exercises in the inductive methods of the physical sciences, if practicable at all, would require a much greater space, and a very different mode of treatment from that which they could receive in this work. For the present, at all events, I must content myself with referring readers to the ample exposition of inductive methods contained in the 3rd, 4th, and 5th books of the *Principles of Science.*

Some readers may perhaps be still inclined to object to the Syllogism, and to deductive logic generally, that it is comparatively worthless, because all new truths are obtained by induction. This doctrine has prevailed with many writers from the time of John Locke to that of John Stuart Mill. But if I have proved in Chapters VI., VII., XI., XII., and in other parts of the *Principles of Science*, that *induction is the inverse operation of deduction*, the supreme importance of syllogistic and other deductive reasoning is not so much restored as explained. In reality the cavillers against the syllogism have never succeeded in the slightest degree in weakening the hold of the syllogism upon the human mind: it was against the nature of things that they should succeed. Their position was as sensible as that of a tutor who should recommend his pupils to begin Mathematics with Compound Division, but on no account to trouble themselves with the obsolete formula of the Multiplication Table. In every point of view, then, a thorough command of deductive processes is the necessary starting-point for any attempt to master more difficult and apparently more important processes of reasoning.

In the composition of the didactic parts of this book, I have tried the experiment of throwing my remarks into the form of answers to assumed; or in many cases actual, examination questions. I cannot call to mind any book in which this mode of treatment has been previously adopted, but it seems to lend itself very readily to the clear exposition of knotty points and difficulties. In spite of much popular clamour against examinations, I maintain that to give a clear, concise, and complete written answer to a definite question or problem is not only the best exercise of mind, but also the best test of ability and training, which can be generally applied.

The Frontispiece contains rough facsimilies of ancient logical diagrams which I copied from the fine MS. of Aristotle's *Organon* in the Ambrosian Library at Milan (L. 93, Superior). During a visit to Italy in 1874, I was much surprised and interested by the multitudes of curious diagrammatic exercises to be found in the logical MSS. of the great public libraries of Italy. The abundance of these diagrams shows that rudimentary logical exercises were very popular in the country where, and at the time when, the dawn of modern science began to break. I estimated that a single MS. in the Biblioteca Communale at Perugia (*Aristotelis de Interpretatione cum Comment.* A 55. Graece. Chart. 1485) contained at least eight hundred such diagrams. Those given in the frontispiece are the most ancient which I could discover. The MS. containing these (among others) is assigned in the printed catalogue to the eleventh or twelfth century, but the librarian was of opinion that it might belong to the tenth century. The figure in the centre shows the Greek original of the familiar Square of Logical Opposition, which has survived to this day (see p. 31). The triangular and lunular figures represent respectively the syllogistic moods Darapti, and (I believe) Datisi.

To the imperfect list of the most recent writings on Symbolical Logic, given in this preface, I am enabled to add at the last moment the important new memoir of Professor C. S. Peirce on the Algebra of Logic, the first part of which is printed in the *American Journal of Mathematics*, vol. iii. (15th September 1880). Professor Peirce adopts the relation of *inclusion*, instead of that of *equation*, as the basis of his system.

BRANCH HILL,
HAMPSTEAD HEATH, N.W.,
3rd October 1880.

NOTE TO EDITION OF 1884

THE present Edition has been printed from the Author's own copy, in which he had marked the few corrections and alterations which have now been made.

HARRIET A. JEVONS.

REFERENCE LIST

OF INITIAL LETTERS SHOWING THE AUTHORSHIP OR SOURCE OF QUESTIONS AND PROBLEMS.

A = PROFESSOR ROBERT ADAMSON, Owens College, Manchester.

B = PROFESSOR ALEXANDER BAIN, University of Aberdeen.

C = Cambridge University. Moral Science Tripos, or College Examination Papers.

D = Dublin University.

E = Edinburgh University. PROFESSOR FRASER.

H = REV. JOHN HOPPUS, formerly Professor of Logic, etc. in University College, London.

I = India Civil Service Examinations.

L = London University, Second B.A., Second B.Sc., M.A., M.D. and D.Sc. Examinations.

M = PROFESSOR THOMAS MOFFET, President and Professor in Queen's College, Galway.

O = Oxford University.

P = PROFESSOR PARK, Queen's College, Belfast, and Queen's former University.

R = PROFESSOR CROOM ROBERTSON, University College, London.

W = WHATELY'S *Elements of Logic.*

CONTENTS

STUDIES IN
DEDUCTIVE LOGIC

CHAPTER I

THE DOCTRINE OF TERMS

INTRODUCTION

1. In accordance with custom, I begin this book of logical studies with the treatment of Terms. Besides being customary, this way of beginning is convenient, because some difficulties which might otherwise be encountered in the treatment of propositions and arguments are cleared out of the way. But the continued study of logic convinces me that this doctrine of terms is really a composite and for the most part extra-logical body of doctrine. It is in fact a survival, derived from the voluminous controversies of the schoolmen.

2. The difficulties of metaphysics, of physics, of grammar, and of logic itself, are entangled together in this part of logical doctrine. Thus, if we take such a term as *colour*, and endeavour to decide upon its logical characters, we should say that it is categorematic, because it can stand as the subject of a proposition; it is positive, because it im-

plies the presence rather than the absence of qualities. But is it abstract or concrete? If concrete, it should be the name of a thing, not of the attributes of a thing. Now colour is certainly an attribute of gold or vermilion; nevertheless, colour has the attribute of being yellow or red or blue. Thus I should say that yellowness is an attribute of colour, and if so, colour is concrete compared with yellowness or blueness, while it is abstract compared with gold or cobalt. If this view is right, abstractness becomes a question of degree.

3. Again, a *relative* term is one which cannot be thought except in relation to something else, the *correlative.* Thus nephew cannot be thought but as the nephew of an uncle or aunt; an instrument cannot be thought but as the instrument to some end or operation. But the question arises, Can anything be thought except as in relation to something else? What is the meaning of a table but as that on which dinner is put? What is a chair but the seat of some person? Every planet is related to the sun, and the sun to the planets. Even meteoric stones moving through empty space are related by gravity to the sun attracting them. All is relative, both in nature and philosophy.

4. As to the distinctions of general, singular, and proper terms, connotative and non-connotative terms, etc., they seem to me to be involved in complete confusion. I have shown in the *Elementary Lessons in Logic* (pp. 41-44) that Proper Names are certainly connotative. There would be an impossible breach of continuity in supposing that, after narrowing the extension of 'thing' successively down to animal, vertebrate, mammalian, man, Englishman, educated at Cambridge, mathematician, great logician, and so forth, thus increasing the intension all the time, the single re-

maining step of adding Augustus de Morgan could remove all the connotation, instead of increasing it to the utmost point. But however this and many other questions in the doctrine of terms may be decided, it is quite clear in any case that this part of logic is ill-suited for furnishing good exercises in reasoning. This ground alone is sufficient to excuse my passing somewhat rapidly and perfunctorily over the first part of logic, and going on at once to the subject of Propositions which offers a wide field for useful exercises. Accordingly, after giving brief definitions of the several kinds of terms, a few answers to questions, and a fair supply of unanswered questions and problems, I pass on to the more satisfactory and prolific parts of logic.

DEFINITIONS AND EXAMPLES

5. A general term is one which can be affirmed, in the same sense, of any one of many (*i.e.* two or more) things.

Examples—Building, front-door, lake, steam-engine.

6. A singular term is one which can only be affirmed, in the same sense, of one single thing.

Examples—Queen Victoria, Cleopatra's Needle, the Yellowstone Park.

7. A collective term is one which can be affirmed of two or more things taken together, but which cannot be affirmed of those things regarded separately or distributively.

Examples—Regiment, century, pair of boots, baker's dozen, book (a collection of sheets of paper).

8. A concrete term is a term which stands for a thing.

Examples—Stone, red thing, brute, man, table, book, father, reason.

9. An abstract term is a term which stands for an attribute of a thing.

Examples—Stoniness, redness, brutality, humanity, tabularity, paternity, rationality.

10. A connotative term is one which denotes a subject and implies an attribute.

Examples—Member of Parliament denotes Gladstone, Sir Stafford Northcote, or any other individual member of parliament, and implies that they can sit in parliament; bird denotes a hawk, or eagle, or finch, or canary, and implies that they have all the attributes of birds.

11. A non-connotative term is one which signifies an attribute only, or (if such can be) a subject only.

Examples—Whiteness denotes whiteness only, an attribute without a subject. John Smith (according to J. S. Mill, and some other logicians) denotes a subject or person only, without implying attributes.

12. Concrete general names are always connotative. Such also are *all* adjectives, without exception. Every adjective is the name of a thing to which it is *added*, and implies that the thing possesses qualities. *Red* is the name of *blood* or of other red thing, and implies that it is red. Redness is the abstract term, the name of the quality redness.

13. A positive concrete term is applied to a thing in respect of its possession of certain attributes; a positive abstract term denotes certain attributes.

> *Examples*—Useful, active, paper, rock; usefulness, activity, rockiness.

14. A negative term is applied to a thing in respect of the absence of certain attributes; if abstract the term denotes the absence of such attributes.

> *Examples*—Useless, inactive, not-paper; uselessness, inactivity.

15. An absolute term is the name of a thing regarded *per se*, or without relation to anything else, if such there can be.

> *Examples*—Air, book, space, water.

16. A relative term is the name of a thing regarded in connection with some other thing.

> *Examples*—Father, ruler, subject, equal, cause, effect.

17. A categorematic term is one which can stand alone as the subject of a proposition.

> *Examples*—Any noun substantive; any adjective, any phrase or any proposition used substantively.

18. A syncategorematic term is any word which can only stand as the subject of a proposition in company with some other words.

> *Examples*—Any preposition, conjunction, adjective used adjectively.

19. Differences of opinion may arise concerning almost every one of the definitions given above, and it would not

be suitable to the purpose of this book to discuss the matter further.

In every case, too, we ought before treating any terms to ascertain clearly that there is no ambiguity about their meanings. An ambiguous term is not one term, but two or more terms confused together, and we should single out one definite sense before we endeavour to assign the logical characteristics. The ambiguity of terms has however been sufficiently dwelt upon in the *Elementary Lessons*, Nos. iv. and vi., and it need not be pursued here.

For the further study of the subject of terms the reader is referred to the *Elementary Lessons*; Mill's *System of Logic*, book i., chapters i. and ii.; Shedden's *Logic*, chapters i. and ii.; Levi Hedge's *Logic*, part ii., chapter i.; Martineau, *Prospective Review*, vol. xxix. pp. 133, etc.; Hamilton's *Lectures on Logic*, vol. iii., lectures viii. to xii.; Woolley's *Introduction to Logic*, part i., chapter i.

QUESTIONS AND ANSWERS

20. Describe the logical characters of the following terms—Equal, equation, equality, equalness, inequality, and equalisation.

Equal is a noun-adjective; concrete, as denoting equal things; connotative, as connoting the attribute of equality; general, positive, relative; and syncategorematic, because it cannot as an adjective form the subject of a proposition.

Equation, noun-substantive, originally abstract, as meaning either equality, or the action of making equal. It is now generally used by mathematicians to denote a pair of quantities affirmed to be equal. It is thus concrete, general, positive, perhaps absolute, and categorematic.

Inequality is a noun-substantive, abstract, singular, negative, and categorematic.

Equalisation means the *action* of making equal, an attribute or circumstance of things, not a thing. It is thus abstract, singular, positive, categorematic.

21. What are the logical characters of the terms, drop of oil, oily, oiliness?

A *drop of oil* being a concrete, finite thing, its name will be concrete, general, positive, relative (as having dropped from a mass of oil), collective as regards the particles of oil, connotative as implying the qualities of oiliness, etc., and categorematic.

Oil is concrete, positive, collective, connotative, and categorematic, like drop of oil, and only differs in not admitting, as regards any one kind of oil, of the plural. It is a case of what I have proposed (*Principles of Science*, p. 28; 1st ed., vol. i. p. 34) to call a *substantial term*, but which I find that Burgersdyk, Heereboord, and the older logicians called a *totum homogeneum*, the parts being of the same name and nature with the whole. (Heereboord, *Synopsis Logicae*, p. 83. See also *Mind*, vol. i. p. 210.)

Oily is a noun-adjective, and is concrete, general, positive, connotative, as denoting oil and implying the attributes of oiliness, doubtfully relative, syncategorematic, 1680.

Oiliness, noun-substantive, abstract, singular, positive, categorematic.

Where distinctions are omitted, it may be understood that they are regarded as inapplicable.

22. Describe the logical characters of the terms—Related, relative, relation, relativeness, relationship, relativity.

I have already dwelt, in the *Elementary Lessons* (p. 25), on the prevalent abuse of the word 'relation,' and other like abstract terms. Nothing is more nearly impossible than to reform the popular use of language; but I will point out once again that relation is properly the abstract name of the connection or bearing of one thing to another, this being an attribute of those things. The things in question are properly said to be 'related,' or to be 'relatives.' Thus, fathers, brothers, sisters, aunts, and cousins, are all relatives —*not relations.* Relationship is an abstract term signifying the attribute of being related; it was invented to replace relation when this was wrongly used as a concrete term. The relationship between a mother and her daughter is simply the relation which exists between two such related persons or relatives. Relativeness is an uncommon term sometimes used to replace the abstract sense of relation, where the case is not one of family relation. Relativity is a further abstract term, probably due to Coleridge, and of which the metaphysicians had better have the monopoly.

CHAPTER II

QUESTIONS AND EXERCISES RELATING TO TERMS

1. DESCRIBE the logical characters of the following terms, classifying them according as they are—

   (*a*) Abstract or Concrete.
   (*b*) General or Singular.
   (*c*) Collective or Distributive.
   (*d*) Positive or Negative.
   (*e*) Absolute or Relative.
   (*f*) Categorematic or Syncategorematic.

Prime Minister	Biped
Institution	Saturn
Copper	Bismarck
Shameful	Monarch
The London Library	Unuseful
Collection	The Times
School Board	Paper
Deaf	Augustus de Morgan
Equation	John Jones
Innumerous	John
Purpose	Triangle
Function	Musicalness
Cousin	Board School
The Absolute	Needlepoint
Black	Representation

Europe	Advocate
Injustice	Being
Brace of partridges	Whale
Dumbness	Lawyer
Planetary System	Time
Classification	Manchester

2. In the case of the following terms distinguish with special care between those which are abstract and those which are concrete—

Nature	Animal	Ethericity
Natural	Animalism	Scarce
Naturalness	Animality	Scarcity
Naturalism	Animalcule	Scarceness
Author	Ether	Truth
Authority	Ethereal	Trueness
Authorship	Etherealness	Verity

3. Investigate the ambiguity of any of the following terms as regards their concrete or abstract character—

Weight	Science
Time	Schism
Intention	Space
Vibration	Relation

4. Supply the abstract terms corresponding to the following concrete terms—

Wood	Conduction
Stone	Atmosphere
Conduct	Alcohol
Witness	Axiom
Equal	Gas
Table	Fire
Boy	Socrates

5. In the case of such of the following terms as you consider to be abstract, name the corresponding concrete terms—

Analysis	Nation
Psychology	Vacuity
Extension	Realm
Production	Folly
Socialism	Evidence

6. Do abstract terms admit of being put in the plural number? Distinguish between the terms which are abstract and concrete in the following list, and at the same time indicate which can in your opinion be used in the plural:—colour, redness, weight, value, quinine, equation, heat, warmth, hotness, solitude, whiteness, paper, space. [c.]

7. Investigate the logical characters and ambiguities of the term *form* in all the following expressions:—a religion of forms; the human form; a form of thought; a school form; a mere form; a printer's form; a form of government; form of prayer; good form; essential form.

8. What error is there in the following descriptions?

Peerless—syncategorematic, general, abstract, positive, relative.

Bacon—equivocal, concrete, general, substantial, positive, relative.

Black—categorematic, abstract, general, negative, absolute.

9. Analyse the following sentences as regards the logical character of each term found in them, distinguishing especially between such as are concrete or abstract, collective or distributive, singular or general—

Logic is the science of the formal laws of thought.

Entre l'homme et le monde il faut l'humanité.

'Art is universal in its influence; so may it be in its practice, if it proceed from a sincere heart and a quick observation. In this case it may be the merest sketch, or the most elaborate imitative finish.'

10. Burton, in his *Etruscan Bologna*, p. 234, uses the abstract term *Etruscanicity*. Is it possible in like manner to make an abstract term corresponding to every concrete one? If so, supply abstracts for the following concretes—

Sir Isaac Newton.	Royal Engineers.
Dictionary.	Postal Telegraph.

11. What logical faults do you detect in the following expressions?—

The standard authorship of modern times.

The three great nationalities of Western Europe.

The legal heir is not necessarily a man's nearest relation.

That unprincipled notoriety Pietro Aretino.

12. Coleridge, in a celebrated note to his *Aids to Reflection*, thus defines an *Idea*: 'An Idea is the *indifference* of the objectively real and the subjectively real: so, namely, that if it be conceived as in the Subject, the idea is an Object, and possesses objective truth; but if in an Object, it is then a Subject, and is necessarily thought of as exercising the powers of a Subject. Thus an Idea, conceived as subsisting in an Object, becomes a *Law*: and a law contemplated subjectively (in a mind) is an *Idea*.'

Analyse the meanings of the terms Idea, Object, Subject, Real, Truth, Law, etc., in the above passage, with respect especially to their concreteness or abstractness. [L.]

13. Name the negative terms which correspond to the following positive terms—

Illumination	Variable
White	Famous
Certain	Notorious
Constant	Valid
Dying	Plenty

14. Name the positive terms which correspond to the following negative or apparently negative terms—

Immensity	Falsehood
Inestimable	Unravelled
Disestablishment	Infamous
Unpleasant	Presuppositionless
Want	Shameless
Unloosed	Empty
Indifferent	Intact
Headless	Ignominious

15. In examining the following list of terms, distinguish, as far as possible, between those which are really negative in form and origin, and those which only simulate the character of negatives—

Annulled	Undespairing
Disannulled	Invalid
Antidote	Headless
Infrequent	Independence
Eclipse	Individual
Undisproved	Indolent
The Infinite	Disagreeable
Impassioned	Despairing
Immense	Infant
Purposeless	Deafness

16. Can you find any examples of terms in the dictionary which are true double negatives? 'Paired,' 'Impaired,' and 'Unimpaired,' may perhaps be affirmed respectively of two things which are equal, unequal, and not unequal. Analyse the meaning of each of the following terms, and show whether it is or is not a true double negative—

Indefeasible	Indefatigable
Uninvalided	Uninjured
Undecomposable	Undecipherable
Undefaceable	Undeformed
Indestructible	Indistinguishable

17. How are the denotation and connotation of a concrete term related to the denotation of the corresponding abstract term?

18. Explain the difference of denotation and connotation with reference to the terms Law, Legislator, Legality, Crime. [L.]

19. Compare the connotation of the following sets of terms—

Abbey Westminster Abbey	Caesar Roman
Mineral Oxide of iron Ore	Road Means of communication Railway

20. Distinguish in the following list such terms as are non-connotative, naming at the same time the logician whose opinion on the subject you adopt—

Virtue	Gladstone
Virtuous	Socrates
The mother of the Gracchi	Barmouth
	The Lord Chamberlain

21. Form a list of twelve purely non-connotative names.

22. What is, if any, the connotation of these terms: Charles the First; Richelieu; John Smith; Santa Maria Maggiore?

23. Try to name half-a-dozen perfectly non-relative names, and then inquire whether they really are non-relative. What is the relation implied or involved in each of the following terms?—

Metropolis	County
Realm	Alphabet
Capital city	Sun

24. Show, by examples, that the division of Names into general and singular does not coincide with the division into abstract and concrete. [L.]

25. What kinds of words can stand as the subject of a proposition, and what kinds are excluded? [O.]

26. Distinguish between the distributive, collective, or singular use of these Latin adjectives of quantity: omnis, omnes, cunctus, cuncti, ullus, quidam, aliquis.

27. What is peculiar about the use of certain terms in the following extracts?—

(1) Frenchmen, I'll be a Salisbury to you.

(2) His family pride was beyond that of a Talbot or a Howard.

(3) In quo quisque artificio excelleret, is in suo genere Roscius diceretur.

(4) 'When foe meets foe.'

28. How does Logic deal with verbs, adverbs, and conjunctions?

29. How many logical terms are there in the following witty epigram? Which and what are they?

What is mind? No matter.
What is matter? Never mind.

30. How many logical terms are there in each of the following sentences? Ascertain exactly how many words are employed in each such term.

(1) The Royal Albert Hall Choral Society's Concert is held in the Albert Hall on the Kensington Gore Estate purchased by the Royal Commissioners of the Great Exhibition of 1851.

(2) 'A name is a word taken at pleasure to serve for a mark which may raise in our mind a thought like to some thought we had before, and which being pronounced to others, may be to them a sign of what thought the speaker had before in his mind.'

31. Words, says Hobbes, are insignificant (that is without meaning), 'when men make a name of two names, whose significations are contradictory and inconsistent: as this name, an incorporeal body.'

The following are a few instances of such apparently self-inconsistent names, and the student is requested to add to the list—

(1) Corporation sole.
(2) Trigeminus.
(3) Manslaughter of a woman.
(4) An invalid contract.
(5) A breach of a necessary law of thought.

32. How would you explain the following apparent absurdities?—

An Act of Parliament (1798-99) prohibited the importation of 'French lawns not made in Ireland.'

Ferguson (*History of Architecture*, vol. ii. p. 233) describes a certain Moabite tower as a 'square Irish round tower.'

33. Are the following terms perfectly univocal or unambiguous, or can you point out any equivocation which is possible in their use?—

Penny	Lecture-Room
Charcoal	Victoria Street
Aluminium	Bible
Second	Monday

34. Trace out and explain the ambiguities which affect any of the following terms—

Organ	Stone	March
Sole	Corn	Mood
Ear	Diet	Mean
Bowl	Perch	Force
Rock (stone)	Bole	Bowl
Rock (bird)	Strait	Straight

35. Draw out complete lists of all the words or expressions which have been developed out of the roots of the following words (see *Elementary Lessons in Logic*, pp. 32-36, and Lesson VI.)—

Post	Logic
Section	Faction
Final	Function
Mission	Decline

CHAPTER III

KINDS OF PROPOSITIONS

1. In this chapter propositions will be described and classed according to the ancient Aristotelian doctrine, in which four principal forms of propositions were recognised, thus tabularly stated :—

	Affirmative.	Negative.
Universal	Symbol = **A** All *X* is *Y*	*Symbol* = **E** No *X* is *Y*
Particular	Symbol = **I** Some *X* is *Y*	Symbol = **O** Some *X* is not *Y*

Singular propositions are to be classed as universal, and indefinite propositions, in which no indication of quantity occurs, must be interpreted at discretion as universal or particular. The student is supposed to be familiar with what the ordinary text-books say upon the subject.

I first give a series of Examples of propositions, with brief comments upon their logical form and peculiarities. A copious selection of exercises is then supplied in the next chapter for the student to treat in like manner.

EXAMPLES

2. 'Books are not absolutely dead things.' **O.**

This proposition is indefinite or pre-indesignate, as Hamilton would call it (*Lectures on Logic*, vol. i. (iii.) p. 244); but, as we can hardly suppose Milton to have thought that all books were living things, I take it to mean 'some books are not, etc.,' that is to say, particular negative.

3. 'The weather is cold.' **A.**

The weather means the present state of the surrounding atmosphere, and may be best described as a singular term, which makes the assertion universal.

4. 'Not all the gallant efforts of the officers and escort of the British Embassy at Cabul were able to save them.' **E.**

At first sight this seems to be a particular negative, like 'Not all that glitters is gold'; but a little consideration shows that 'gallant efforts' is a collective whole, the efforts being made in common, and therefore either successful or unsuccessful as a whole. The meaning then is, 'The whole of the gallant efforts, etc., were not able to save the men.' It is a universal negative.

5. 'One bad general is better than two good ones.' **A.**

This saying of Napoleon looks at first like a particular or even a singular proposition; but the 'one bad general' means not any definite one, but 'any one bad general' acting alone.

6. 'No non-metallic substance is now employed to make money.' **E.**

The subject is a negative term, and the proposition might be stated as 'All non-metallic substances are not any of those employed to make money.'

7. 'Multiplication is vexation.'

If *all* multiplication is so, this is **A**; there are certainly other causes of vexation.

8. 'Wealth is not the highest good.' **E.**

Affirmatively, wealth is one of the things which are not the highest good.

9. 'Murder will out.' **A.**

Like most proverbs, this is an unqualified universal proposition; its material truth may be doubted.

10. 'A little knowledge is a dangerous thing.' **A.**

This looks like a particular affirmative, but is really **A**, as meaning that 'any small collection of knowledge is, etc.'

11. 'All these claims upon my time overpower me.' **A.**

Dr. Thomson points out (*Outline*, 5th ed., p. 131) that *all* is here clearly collective.

12. 'The whole is greater than any of its parts.' **A.**

Though apparently singular, this is really a general axiom, meaning 'any whole is greater, etc.'

13. 'No wolves run wild in Great Britain at the present day.' **E.**

14. 'Who seeks and will not take, when once 'tis offered, shall never find it more.' **E.**

This seems to be a compound proposition, but the subject is, 'Any one who is seeking, but has not taken when once it was offered.'

15. 'The known planets are now more than a hundred in number.' **A.**

Clearly a collective singular affirmative proposition, and therefore universal. Of course the planets separately could not have the predicate here affirmed.

16. 'Figs come from Turkey.' **I.**

Indesignate; that is to say, we cannot assume without express statement that it is intended to say, 'All figs come from Turkey.'

17. 'Xanthippe was the wife of Socrates.' **A.**

18. 'No one is free who is enslaved by his appetites.' **E.**

19. 'Certain Greek philosophers were the founders of logic.' **A.**

Apparently **I**; but if 'certain' means a certain definite group of men, each of whom was essential in his time, the proposition becomes collective and singular, hence universal.

20. 'Comets are subject to the law of gravitation.' **A.**

Indefinite affirmative; but in a matter of such universality it may be interpreted as **A.**

21. 'Democracy ends in despotism.' **I.**

Again indefinite; but as referring to matter in which no rigorous laws have been detected it should be interpreted particularly.

22. 'Men at every period since the time of Aristotle have studied logic.' **I.**

Obviously particular as regards 'men.'

23. 'Few men know how little they know.' **O.**

That is, 'Most men do not know, etc.' Hence **O.**

24. 'Natura omnia dedit omnibus.' **A.**

Singular affirmative, because *natura* is a singular term. The assertion is one of Hobbes', and is thoroughly ambiguous as regards *omnia* and *omnibus*, which might be capable either of collective or distributive meaning. No doubt, however, the meaning is that Nature did not assign anything to any particular person; if so, both must be taken collectively.

25. 'There are many cotton-spinners unemployed.' **I.**

Really a kind of numerical assertion; but if to be classed at all, it must be **I**, 'many' being only a part of 'all.'

26. 'A few Macedonians vanquished the vast army of Darius.' **A.**

Collective singular affirmative, because the *few* of course acted together. It is a question whether the predicate is not also singular.

27. 'True Faith and Reason are the soul's two eyes.' **A.**

Collective singular.

28. 'A perfect man ought always to be busy conquering himself.' **A.**

'All' perfect men ought, etc.

29. 'A truly educated man knows something of everything and everything of something.' **A.**

There seems to be two predicates, and hence a compound sentence; but this is not the case, because the truly educated man must know both.

30. 'Some comets revolve in hyperbolic orbits.' **I.**

Particular affirmative as it stands.

31. 'The dividends are paid half-yearly.' **A.**

'The dividends' includes all so known.

32. 'Ου τὸ μεγα εὐ ἐστι, το δ' εὐ μεγα.' **O** and **A.**

This must mean that not all great things are good (**O**), but that all good things are great (**A**). There are three classes of things—great and good; great and not-good; not-great and not-good.

33. 'It is force alone which can produce a change of motion.' **A.**

It = what can produce, etc. The meaning is, Whatever produces a change of motion is some kind of force; but there is no assertion that force = whatever produces, etc.

34. 'We have no king but Caesar.'

As it stands, **A**; but the meaning conveyed implies that 'Caesar is our king'; 'Nobody who is not Caesar is our king.'

35. 'It is true that what is settled by custom though it be not good, yet at least it is fit.'

Complex; three propositions in all.

36. 'God did not make man, and leave it to Aristotle to make him rational.'

A simple and a singular negative proposition; the 'not' applies to all that follows conjunctively, for of course Locke could not have intended to assert that 'God did not make man.' **E.**

37. 'Dublin is the only city in Europe, save Rome, which has two cathedrals.'

Compound sentence implying three propositions, namely—

Dublin has two cathedrals. **A.**

Rome has two cathedrals. **A.**

All European cities, not being Dublin and not being Rome, have not two cathedrals. **E.**

38. 'The affections are love, hatred, joy, sorrow, hope, fear, and anger.'

Really a disjunctive proposition. Affection is either love, or hatred, or, etc. This implies that love is an affection, hatred is an affection, etc.

CHAPTER IV

EXERCISES IN THE DISCRIMINATION OF PROPOSITIONS

1. EXAMINE each of the following propositions, and point out in succession—

(*a*) Which is the subject.
(*b*) Which is the predicate.
(*c*) Whether the proposition is affirmative or negative.
(*d*) Whether it appears to be universal or particular.
(*e*) Whether there is ambiguity or other peculiarity in the proposition.

(1) All foraminifera are marine organisms.
(2) They never pardon who have done the wrong.
(3) Great is Diana of the Ephesians.
(4) No mammalia are parasites.
(5) Non progredi est regredi.
(6) Not every one can integrate a differential equation.
(7) All, all are gone, the old familiar faces.
(8) He that is not for us is against us.
(9) Ἄριστον μὲν ὕδωρ.
(10) Men mostly hate those whom they have injured.
(11) Old age necessarily brings decay.
(12) Nothing morally wrong is politically right.
(13) What I have written I have written.
(14) It is not good for man to be alone.

(15) A certain man had a fig-tree.
(16) Χαλεπὰ τὰ καλά.
(17) There's something rotten in the state of Denmark.
(18) To be or not to be, that is the question.
(19) Ye are my disciples, if ye do all I have said unto you.
(20) Possunt qui posse videntur.
(21) There can be no effect without a cause.
(22) Rien n'est beau que le vrai.
(23) Pauci laeta arva tenemus.
(24) All cannot receive this saying.
(25) Fain would I climb, but that I fear to fall.
(26) There's not a joy the world can give like that it takes away.
(27) Not to know me argues thyself unknown.
(28) Two blacks won't make a white.
(29) Few men are free from vanity.
(30) He that fights and runs away may live to fight another day.
(31) We are what we are.
(32) There is none good but one.
(33) Two straight lines cannot inclose space.
(34) Better late than never.
(35) Cruel laws increase crime.
(36) Omnes omnia bona dicere.
(37) Le génie n'est qu'une plus grande aptitude à la patience.
(38) Whosoever is delighted in solitude is either a wild beast or a god.
(39) Summum jus summa injuria.
(40) Non omnes moriemur inulti.
(41) Haud ignara mali miseris succurrere disco.
(42) Familiarity breeds contempt.
(43) Some politicians cannot read the signs of the times.

(44) Only the ignorant affect to despise knowledge.

(45) Recte ponitur; vere scire esse per causas scire.

(46) Only Captain Webb is able to swim across the Channel.

(47) Some books are to be read only in parts.

(48) E pur si muove.

(49) Civilisation and Christianity are coextensive.

(50) Some men are not incapable of telling falsehoods.

(51) Sunt nonnulli acuendis puerorum ingeniis non inutiles lusus.

(52) All is not true that seems so.

(53) Me miserable.

(54) The Claimant, Arthur Orton, and Castro are in all probability the same person.

(55) The three angles of a triangle are necessarily equal to two right angles.

(56) Many rules of grammar overload the memory.

(57) Nullius exitium patitur natura videri.

(58) Summae artis est occultare artem.

(59) Wonderful are the results of science and industry in recent years.

(60) Love is not love which alters when it alteration finds.

(61) A healthy nature may or may not be great; but there is no great nature that is not healthy.

(62) Quas dederis solas semper habebis opes.

(63) Quod volunt, id credunt homines.

(64) *Πᾶσα σὰρξ οὐ δικαιωθήσεται.*

(65) Antiquitas seculi, juventus mundi.

(66) That would hang us, every mother's son.

(67) Men in great place are thrice servants.

(68) Justice is ever equal.

(69) A friend should bear a friend's infirmities.

(70) Men are not what they were.

(71) The troops took one hour in passing the saluting point.
(72) Nemo mortalium omnibus horis sapit.
(73) Fugaces labuntur anni.
(74) Αὐτὸς ἐγὼ εἰμί.
(75) Communia sunt amicorum inter se omnia.
(76) Dictum sapienti sat est.
(77) The Romans conquered the Carthaginians.
(78) The fear of the Lord, that is wisdom.
(79) To live in hearts we leave below is not to die.
(80) 'Tis only noble to be good.
(81) Dum spiro spero.

2. In looking over the following list of propositions distinguish between those which have a distributive and those which have a collective subject.

(1) All the asteroids have been discovered during the present century.
(2) All Albinos are pink-eyed people.
(3) The facts of aboriginal life seem to indicate that dress is developed out of decorations.
(4) Non omnes omnia decent.
(5) Dirt and overcrowding are among the principal causes of disease.
(6) Omnes apostoli sunt duodecim.
(7) Many artisans are unemployed.
(8) The side and diagonal of a square are incommensurable.
(9) Omnis homo est animal.
(10) Nihil est ab omni parte beatum.

3. Ascertain exactly how many distinct assertions are made in each of these sentences, and assign the logical characters of the propositions.

(1) 'Tis not my profit that doth lead mine honour: mine honour, it.

(2) True, 'tis a pity; pity 'tis, 'tis true.

(3) Hearts, tongues, figures, scribes, bards, poets, cannot think, speak, cast, write, sing, number, ho! his love to Antony.

(4) A horse, a horse! my kingdom for a horse.

(5) Istuc est sapere, non quod ante pedes modo est videre: sed etiam illa, quae futura sunt, prospicere.

(6) Virtue consists neither in excess nor defect of action, but in a certain mean degree.

(7) The glories of our blood and state are shadows, not substantial things.

(8) 'To gild refined gold, to paint the lily,
To throw a perfume on the violet,
To smooth the ice, or add another hue
Unto the rainbow, or with taper light
To seek the beauteous eye of heaven to garnish
Is wasteful and ridiculous excess.'

(9) All places that the eye of heaven visits,
Are to a wise man ports and happy havens.

(10) The age of chivalry is gone, and the glory of Europe extinguished for ever.

(11) Poeta nascitur, non fit.

(12) Not all speech is enunciative, but only that in which there is truth or falsehood.

(13) Devouring Famine, Plague, and War,
Each able to undo Mankind,
Death's servile emissaries are.

(14) Many are perfect in men's humours, that are not greatly capable of the real part of business, which is the constitution of one that hath studied men more than books.

(15) Vivre, ce n'est pas respirer, c'est agir.

(16) Justice is expediency, but it is expediency speaking by general maxims, into which reason has concentrated the experience of mankind.

(17) Men, wives, and children, stare, cry out, and run as it were doomsday.

4. Distinguish so far as you can between the propositions in the following list which are to you explicative and ampliative. (See *Elementary Lessons*, pp. 68-69. Thomson's *Outline of the Necessary Laws of Thought*, § 81.)

(1) Homer wrote the Iliad and Odyssey.

(2) A parallelopiped is a solid figure having six faces, of which every opposite two are parallel.

(3) The square on the hypothenuse of a right-angled triangle is equal to the sum of the squares on the sides containing the right angle.

(4) The swallow is a migratory bird.

(5) Axioms are self-evident truths.

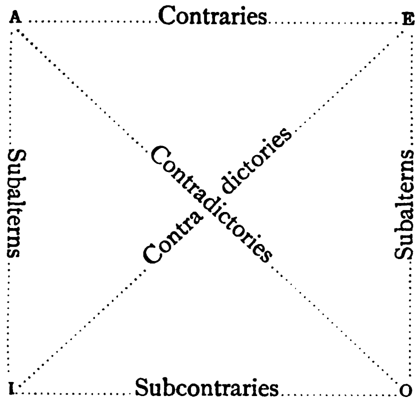
5. Classify the following signs of logical quantity according as they are generally used to indicate universality, affirmative or negative, or particularity, affirmative or negative—

Several, none, certain, few, ullus, nullus, nonnullus, not a few, many, the whole, almost all, not all.

CHAPTER V

CONVERSION OF PROPOSITIONS, AND IMMEDIATE INFERENCE

1. THE student is referred to the *Elementary Lessons in Logic*, or to other elementary text-books, for the common rules of conversation and immediate inference, but, for the sake of easy reference, the ancient square of opposition is given below.



All the relations of propositions and the methods of inference applying to a single proposition will be found fully exemplified and described in the following questions and answers.

2. It appears to be indispensable, however, to endeavour to introduce some fixed nomenclature for the relations of propositions involving two terms. Professor Alexander Bain has already made an innovation by using the name *obverse*, and Professor Hirst, Professor Henrici and other reformers of the teaching of geometry have begun to use the terms converse and obverse in meanings inconsistent with those attached to them in logical science (*Mind*, 1876, p. 147). It seems needful, therefore, to state in the most explicit way the nomenclature here proposed to be adopted with the concurrence of Professor Robertson.

Taking as the original proposition 'all *A* are *B*,' the following are what we may call the *related propositions*—

INFERRIBLE.

Converse. Some *B* are *A*.
Obverse. No *A* are not *B*.
Contrapositive. No not *B* are *A*,
or, all not *B* are not *A*.

NON-INFERRIBLE.

Inverse. All *B* are *A*.
Reciprocal. All not *A* are not *B*.

It must be observed that the converse, obverse, and contrapositive are all true if the original proposition is true. The same is not necessarily the case with the *inverse* and *reciprocal*. These latter two names are adopted from the excellent work of Delbœuf, *Prolégomènes Philosophiques de la Géométrie*, pp. 88-91, at the suggestion of Professor Croom Robertson. (*Mind*, 1876, p. 425.)

QUESTIONS AND ANSWERS

3. Give all the logical opposites of the proposition, 'All metals are conductors.'

This is a universal affirmative proposition, having the symbol **A**. By its *logical opposites* we mean the corresponding propositions in the forms **E**, **I**, and **O**, which have the same subject and predicate, and are related to it respectively as its contrary, contradictory, and subaltern, in the way shown in the Logical Square (p. 31) and explained in many Manuals. These opposite propositions may be thus stated—

Subaltern	(**I**)—Some metals are conductors.
Contradictory	(**O**)—Some metals are not conductors.
Contrary	(**E**)—No metals are conductors.

The first of these (**I**) may be inferred from the original; the other two (**O** and **E**), so far from being inferrible, are inconsistent with its truth.

4. Given that a particular negative proposition is true, is the following chain of inferences correct?—**O** is true, **A** is false, **I** is false, and therefore **E** is true. If so, the truth of **O** involves the truth of **E**.

There is a false step in this argument; for the falsity of **A** does not involve the falsity of **I**. It may be (and is materially false) that 'all men are dishonest'; but it nevertheless may remain true that 'some men are dishonest.' Observe, then, that the falsity of **A** does not involve the truth of **I**, nor does the truth of **I** involve the truth or falsity of **A**. But the truth of **A** necessitates that of **I**. As stated in the *Elementary Lessons* (p. 78), 'Of subalterns, the particular is true if the universal be true: but the universal may or may not be true when the particular is true.'

5. How do you convert universal affirmative propositions?

They must be converted by limitation or *per accidens*, as it is called, that is to say, while preserving the affirmative quality, the quantity of the proposition must be limited from universal to particular. Thus **A** is converted into **I**, as in the following more or less troublesome instances, the Convertend standing first and the Converse second in each pair of propositions:—

All organic substances contain carbon.
Some substances containing carbon are organic.

Time for no man bides.
Something biding for no man is time.

The poor have few friends.
Some who have few friends are poor.

A wise man maketh more opportunities than he finds.
Some who make more opportunities than they find are wise men.

They are ill discoverers who think there is no land, when they can see nothing but water.
Some ill discoverers think there is no land, etc.

Great is Diana of the Ephesians.
Some great being is Diana of the Ephesians.

Warm-blooded animals are without exception air-breathers.
Air-breathers are (with or without exception) warm-blooded animals.

6. How would you convert 'Brutus killed Caesar?'

The strictly logical converse is 'Some one who killed Caesar was Brutus.' For, though a man can only be killed once, and Brutus is distinctly said to be the killer, yet in formal logic we know nothing of the *matter*, and Caesar might have been killed on other occasions by other persons. An absurd illustration is purposely chosen in the hope that it may assist to fix in the memory the all-important truth that in logic we deal not with the matter.

7. How do you convert particular affirmative propositions?

To this kind of proposition simple conversion can be applied; that is to say, the converse will preserve both the quantity and the quality of the convertend. In other words, **I** when converted gives another proposition in **I**; thus either of the following pairs is the simple converse of the other:—

Some dogs are ferocious animals.
Some ferocious animals are dogs.

Some men have not courage to appear as good as they are.
Some, who have not courage to appear as good as they are, are men.

Some animals are amphibious.
Some amphibious beings are animals.

8. How do you convert universal negative propositions?

These also are converted *simply*, giving another universal negative proposition. **E** gives **E**. The reason is that both the terms of **E** are distributed; a universal negative asserts complete separation between the whole of the subject and the whole of the predicate. 'No man is a tailed animal' asserts that not any one man is found anywhere in the class of tailed animals. Hence it follows evidently that no one being belonging to the class of tailed animals is found in the class of men, which result we assert in the simple converse proposition, 'no tailed animal is a man.' Further examples of the same mode of conversion are given below.

No virtue is ultimately injurious.
No ultimately injurious thing is a virtne.

No wise man runs into heedless danger.
No one who runs into heedless danger is a wise man.

People will not look forward to posterity who never look backward to their ancestors.
People never look backward to their ancestors who will not look forward to posterity.

Whatever is insentient is not an animal.
Whatever is an animal is not insentient.

9. How do you convert particular negative propositions?

Difficulty arises about this question, because the first rule of conversion tells us to preserve the quality of the proposition; the converse accordingly should be negative. But a negative proposition always distributes its predicate, because a thing excluded from a class must be excluded from every part of the class. Now the subject of **O** being particular and indefinite, it cannot stand as a distributed predicate. It is still possible to say with material truth, 'some men are not soldiers'; but converted this gives the absurd result, 'all soldiers are not men'; or, 'no soldiers are men.' Even if we insert the mark of quantity 'some' before the predicate, and say, 'all soldiers are not *some* men,' we must remember that 'some' is perfectly indefinite, and may include *all.* The question will be more fully discussed further on, but, so far as I can see, the particular negative proposition, so long as it remains negative and indefinite in meaning, is incapable of conversion. This fact constitutes a blot in the ancient logic.

Nevertheless the proposition **O** is capable of giving a converse result when we change it into the equivalent affirmative proposition. If 'some men' are excluded from the class 'soldiers,' they are necessarily included in the class non-soldiers, or, 'some men are non-soldiers.' This is a proposition in **I**, and by simple conversion, as already described, gives a converse also in **I**, 'some non-soldiers are men.' As further examples take—

Some dicotyledons have not reticulate leaves.
Some plants with non-reticulate leaves are dicotyledons.

Some crystals are not symmetrical.
Some unsymmetrical things are crystals.

All men have not faith.
Some who have not faith are men.

Not every one that saith unto me, Lord, Lord, shall enter into the Kingdom of Heaven.
Some who shall not enter into the Kingdom of Heaven say unto me, Lord, Lord.

10. How do you convert singular propositions?

Singular propositions, being those which have a singular term as subject, may be divided into two classes, according as the predicate is a singular or a general term. (See Karslake, 1851, vol. i. p. 54.) The former will always be converted simply, one single thing being identified with the same under another name, as in 'Queen Victoria is the Duchess of Lancaster,' converted into 'the Duchess of Lancaster is Queen Victoria.' Simple conversion will also apply if the predicate be a general term, provided that the proposition be negative so as to distribute this term. Thus, 'St. Albans is not a great city' becomes 'no great city is St. Albans.' But if the predicate be general and undistributed, as in an affirmative singular proposition, then we must convert *per accidens*, and limit the new subject to some or even one significate of the general term. Examples of each case follow:—

The better part of valour is discretion.
Discretion is the better part of valour.

Time is the greatest innovator.
The greatest innovator is time.

London is the greatest of all cities.
The greatest of all cities is London.

London is not a beautiful city.
No beautiful city is London.

Le style est l'homme même.
L'homme même est le style.

All the allied troops fought courageously.
Some who fought courageously were the allied troops.

Mercy but murders, pardoning those that kill.
Something which murders is mercy, pardoning those that kill.

Not all the figures that Babbage's calculating machine could run up, would stand against the general heart.
Something which would not stand against the general heart is all the figures (collectively) that Babbage's machine could run up.

11. Show how to convert the propositions—

(1) 'All mathematical works are not difficult.'
(2) 'All equilateral triangles are equiangular.'
(3) 'No triangle has one side equal to the other two.'

The first proposition, as it stands, is ambiguous, for it looks like the universal negative, 'no mathematical works are difficult.' But, according to custom, we may interpret it as meaning that 'not all mathematical works are difficult,' or 'some mathematical works are not difficult,' a proposition in the form **O**. This cannot be converted simply, as already explained (p. 36), because we must preserve the negative quality, and 'all (or some) difficult things are not mathematical works' being negative would distribute its predicate 'mathematical works.' We can, however, make **O** into **I**, 'some mathematical works are not-difficult things,' and we can convert this simply into 'some not-difficult things are mathematical works.'

Proposition (2), as it stands, is in **A**, and can only be logically converted by limitation into 'some equiangular triangles are equilateral.' Geometrically it could easily be shown that the *inverse* proposition 'all equiangular triangles are equilateral,' is also true; but we must of course not allow knowledge of the matter in question to influence us in logical deduction, and the inverse proposition cannot be inferred from the original.

Number (3) is a universal negative, and must be converted simply into 'Nothing having one side equal to the other two is a triangle'; but there is something paradoxical about this result, which the student is recommended to investigate.

12. Convert 'Life every man holds dear.'

This is an example given in the *Elementary Lessons* (p. 304). Students have variously converted it into—

Life is held dear by every man.
Some life is held dear, etc.
No man holds death dear (!)

and so forth. But it ought surely to be easy to see that the grammatical object is transposed, 'life' being the object of 'holds dear.' The statement is that 'every man holds life dear,' and is explicitly a universal affirmative proposition, to be converted by limitation into 'some who hold life dear are men.'

13. Convert the proposition 'It rains.'

What is it that rains? What is 'it'? Surely the environment, or more exactly the atmosphere. The proposition then means 'the atmosphere is letting rain fall.' The converse will therefore be 'something which is letting rain fall is the atmosphere.' But in this and many other cases the Aristotelian process of conversion by limitation gives a meaningless if not absurd result.

14. Convert the proposition 'He jests at scars who never felt a wound.'

This is the 8th example on p. 304 of the *Elementary*

Lessons, and has elicited from time to time sor efforts at conversion, such as—

> Some jests at scars are made by one who never felt a wound.
> Scars are jested at by him who, etc.
> Some scars jest at him who never felt a wound. (*sic.*)
> Some scars are jests to one who, etc.

The subject of the proposition is of course 'he who never felt a wound,' and the proposition asserts that he thus described 'jests at scars.' As there is no limitation of quantity we may take the subject as universal; and, although there is negation within the subject, the copula is affirmative, and the proposition is in the form **A**. It is thus converted by limitation into 'some who jest at scars are persons who have never felt a wound.'

15. Convert the proposition '*P* struck *Q*.'

To this simple question I have got answers that, since *P* is distributed, and *Q* undistributed, we must convert by limitation, getting 'some *Q* struck *P*'; or by contraposition 'some not-*Q* struck not-*P*.' Such blunders and nonsense arise from failing to notice that 'struck' is not a simple logical copula. There is, of course, a relation between *P* and *Q*; but as regards *P*, the proposition simply asserts that '*P* is a person who struck *Q*,' possibly not the only one. Hence the converse by limitation is 'some person who struck *Q* is *P*.'

Not a few examinees would at once convert '*P* struck *Q*' into '*Q* struck *P*,' but this, although very likely to happen materially, is not logically necessary.

16. What is the *obverse* of the proposition 'All metals are elements'?

The *obverse* is a new term introduced by Professor Alexander Bain, and its meaning is thus described by him in his *Deductive Logic*, pp. 109, 110. 'In affirming one thing, we must be prepared to deny the opposite: "the road is level," "it is not inclined," are not two facts, but the same fact from its other side. This process is named OBVERSION.' He proceeds to point out that each of the four propositional forms, **A**, **I**, **E**, **O**, admits of an obverse. 'Every X is Y' becomes 'no X is not-Y.' 'Some X is Y' becomes 'some X is not not-Y.' 'No X is Y' becomes 'all X is not-Y.' 'Some X is not Y' becomes 'some X is not-Y.' Accordingly the obverse of the proposition above will be, 'No metals are not elements.'

Professor Bain goes on to describe what he calls 'Material Obversion,' justified only on an examination of the matter of the proposition. Thus from 'warmth is agreeable,' he infers, after examination of the subject-matter, that 'cold is disagreeable.' 'If knowledge is good, ignorance is bad.' I feel sure, however, that this mixing up of so-called material obversion with formal obversion is likely to confuse people altogether. Indeed, Mr. Bain is himself confused, for he cites, 'I don't like a curving road, because I like a straight one,' as a childish reason, 'being no reason at all, but the same fact in obverse.' Now, if there is any relation at all between these two propositions, it is certainly a case of material obversion; but in reality they do not express the same fact at all. The formal obverse of 'I like a straight road,' is 'I am not one who does not like a straight road.' We might perhaps infer, 'I do not dislike a straight road'; but there is clearly no reference to curved roads at all.

While accepting the new term *obversion* in the sense of *formal obversion*, I must add that students have begun to use it with the utmost laxity, confusing the obverse with the converse, the contrapositive, etc. To prevent logical nomenclature from falling into complete chaos, it seems to be indispensable to choose convenient names for the simpler relations of propositional forms, as attempted above (p. 32), and to adhere to them inflexibly.

17. What is conversion by contraposition? Give the contrapositive of 'All birds are bipeds.'

There is nothing which I have found so difficult in teaching logic as to get the student to comprehend and remember this process of *contraposition*; particular attention is therefore requested to the above question.

Having a proposition in **A**, we get its contrapositive by taking the negative of its predicate, and affirming of this as a subject the negative of the original subject. Thus, if 'all *X*s are *Y*s,' we take all not-*Y*s as a new subject, and affirm of them that they are all not-*X*s, getting the proposition 'all not-*Y*s are not-*X*s,' which is either **A** or **E**, according as we do or do not join the negative particle to the predicate *X*. Accordingly the contrapositive of the proposition 'all birds are bipeds' will be 'all that are not bipeds are not birds.'

It is one thing to obtain the contrapositive, another thing to see that it may be inferred from the premise. The late Professor De Morgan used to hold that the act of inference is a self-evident one, and needs no analysis; but the process may certainly be analysed. Thus we may obvert the premise 'All *X*s are *Y*s,' obtaining 'No-*X*s are not *Y*s,' which is a proposition in **E**, and then convert simply into 'No not-*Y*s are *X*s,' also in **E**, or else 'All not-*Y*s are not *X*s.' The contrapositive, then, is the converse of the obverse.

We may also prove the truth of the contrapositive indirectly; for what is not-*Y* must be either *X* or not-*X*;

but if it be *X* it is by the premise also *Y*, so that the same thing would be at the same time not-*Y* and also *Y*, which is impossible. It follows that we must affirm of not-*Y* the other alternative, not-*X*. (See Chapter XXI. below; also *Principles of Science*, pp. 83, 84; first ed., vol. i. pp. 97, 98.)

18. Give the converse of the contrapositive of the proposition 'All vegetable substances are organic.'

As learnt from the last question, the contrapositive is 'All not-organic substances are not vegetable substances.' We may take this to be equivalent to 'No inorganic substances are vegetable substances' (**E**), the simple converse of which is 'No vegetable substances are inorganic substances,' the obverse of the premise. But, if we treat the contrapositive as a universal affirmative proposition, thus, 'All inorganic substances are non-vegetable substances,' we must convert by limitation, getting 'Some non-vegetable substances are inorganic,' which is the subaltern of the obverse, and cannot by any process of inference lead us back to the original. Conversion by limitation is easily seen to be a faulty process which always occasions a loss of logical force.

As we shall afterwards observe, this kind of conversion introduces a new term, namely the indeterminate adjective 'some,' so that the inference is not really confined to the terms of the original premise. Although we may not be able to dispense entirely with the word, owing to its employment in ordinary discourse, we shall ultimately eliminate it from pure formal logic, and relegate it to the branch of numerical logic.

19. Take the following proposition, 'all water contains air'; convert it by contraposition: change the result into an affirmative proposition, and convert.

To show the need of more careful logical training than has hitherto been common, even in the great Universities, I give a few specimens of answers which I received to the above question. The contrapositive of the proposition was variously stated, as

All air does not contain all water.
All air is not contained in water.
All not-air is not a thing contained by not-water.
Some air is not contained in water.
Some not-air contains no water.
All not-air contains water.

The logicians who drew these inferences then proceeded by simple conversion to get such results as the following:—

Some water is not without some air.
No water contains not some air.
No water contains no air.

One too clever student inferred that 'All or every vacuum is a void of water,' which he converted, simply indeed, into

'Every void of water is a vacuum'!

An examiner in logic is sometimes forced to believe that there is a void in the brains of an examinee; but the absence of any sufficient training in logical work is more often the cause of the lamentable results shown above. In any case it seems impossible to agree with De Morgan that contraposition is a self-evident process.

These absurd answers are mainly due to the failure to observe that in the proposition 'All water contains air,' the two words 'contains air,' form the *grammatical predicate*, comprehending both the logical predicate and the logical copula. Logically then the proposition is 'All water is containing air,' or 'All water is what contains air.' The contrapositive then is 'All that does not contain air is not water.' Uniting the negative particle to the predicate 'water,' and converting by limitation, we obtain 'Some not-water is what does not contain air.'

20. Describe the logical relations, if any, between each of the following propositions and each other—

(1) All organic substances contain carbon.

(2) There are no inorganic substances which do not contain carbon.

(3) Some inorganic substances do not contain carbon.

(4) Some substances not containing carbon are organic.

Of these, (1) is a universal affirmative, the contrapositive of which is 'All substances not containing carbon are inorganic substances.' Hence the converse by limitation of this contrapositive is, 'Some inorganic substances are substances not containing carbon,' equivalent to (3).

Proposition (2) is the obverse of 'All inorganic substances contain carbon,' which is the contradictory of (3).

To obtain (4) we must take the contrary of (1), that is, no organic substances contain carbon, express it in the affirmative form, 'All organic substances are substances not containing carbon,' and then convert it by limitation.

21. Take any proposition suitable for the purpose, convert it by contraposition, convert it again *simpliciter*, change the result into an affirmative proposition, and show that you may regain the original proposition. [C.]

The most suitable kind of proposition for the purpose will be a universal affirmative, such as

(1) All birds are bipeds.

The contrapositive may be stated in the form of **E**.

(2) No not-bipeds are birds.

Which is converted simpliciter into **E**, the obverse of (1).

(3) No birds are not-bipeds.

When thrown into the affirmative form by a second obversion, the last becomes

(4) All birds are not-not-bipeds.

As double negation destroys itself, this is equal to (1). Notice that the obverse of the obverse is the original.

22. Give the converse of the contradictory of the proposition, 'There are no coins which are not made of metal.'

The premise is stated in a complex form with double negation; it means 'No coins are not made of metal,' which is the obverse of 'All coins are made of metal' (**A**). The contradictory, as shown in the square of opposition (p. 31), is a proposition in **O**, namely, 'Some coins are not made of metal,' which can be converted only by negation, that is, by joining the negative particle to the predicate, thus: 'Some coins are not-made-of-metal,' whence by simple conversion 'Some things not-made-of-metal are coins' the answer required.

23. (1) All crystals are solid.
(2) Some solids are not crystals.
(3) Some not-crystals are not solids.
(4) No crystals are not-solids.
(5) Some solids are crystals.
(6) Some not-solids are not crystals.
(7) All solids are crystals.

Assign the logical relation, if any, between each of these propositions and the first of them.

Proposition (1) is a universal affirmative (**A**); its simple obverse is (4); its converse by limitation is (5); the subcontrary of this converse is (2). In order to obtain (6) we must take the contrapositive of (1), namely, 'All not-solids are not crystals,' the subaltern of which is (6); and converting (6) by negation we get (3). Again, (7) is the *inverse*, but is not inferrible from (1). We may further say that (4) can be inferred from (1), and is exactly equivalent in logical force to it; (5) and (6) can be inferred, but are not equivalent to the original; (2) cannot be inferred from (1), but is not inconsistent with its truth.

24. What information about the term not-*A* can we derive from the premise 'All *A*s are *B*s'?

This question, though apparently a very simple one, does not admit of a very simple answer; it is important in a theoretical point of view. It may be said on the one hand, that as the proposition only affirms of all *A*s that they are *B*s, this tells us nothing about things excluded from the class *A*. Thus what is not-*A* may be *B*, or it may not be *B*, without any interference from the premise. This is quite true. About Not-*A* *universally* we may infer nothing.

But, on the other hand, if we convert the proposition 'all

*A*s are *B*s' by contraposition (p. 43), we get 'all not-*B*s are not *A*s.' Uniting the negative particle to the predicate, we have 'All not-*B*s are not-*A*s,' whence, by limited conversion, we infer some not *A*s are not-*B*s. In this result we must interpret *some* as meaning, *one at least, it may be more or even all.* We shall recur to this question in a subsequent chapter.

25. Assuming that no organic beings are devoid of carbon, what can we thence infer respectively about beings which are not organic, and things which are not devoid of carbon?

The premise 'No organic beings are devoid of carbon' is a universal negative proposition, and does not directly give information about beings which are not organic, and beings which are not devoid of carbon. But, if we join the negative particle to the predicate, we get 'All organic beings are not-devoid-of-carbon,' whence, by limited conversion, 'Some things not devoid of carbon are organic,' which answers the second part of the question.

Again, converting by contraposition, we learn that 'All things not-not-devoid of carbon are not organic beings'; in other words, 'All things devoid of carbon are not organic beings,' a result which may be obtained perhaps more clearly by converting the original premise simply, thus, 'No things devoid of carbon are organic beings,' or 'All things devoid of carbon are not organic beings.' Conversion by limitation then yields 'Some things not organic beings are devoid of carbon,' which is the answer to the first part of the question. This result is the same as that obtained in the last question, and the same remarks apply.

26. What information about the term Solid Body can we derive from the proposition, 'No bodies which are not solids are crystals'?

This question differs from the last only in being put in a more involved form. The premise when more simply stated becomes 'All not solids are not crystals,' the contrapositive of 'All crystals are solids,' and limited conversion gives 'Some solids are crystals.'

27. Nihil potest placere, quod non decet. Convert this proposition, (1) simply, (2) by contraposition; and show by what logical processes we can pass back from the contrapositive to the original. [C.]

This premise (from Quinctilian, c. xi. 65) equals, Nihil quod non decet, potest placere; nothing which is unbecoming can please. Being a universal negative, **E**, it can be converted simply into 'Nothing which can please is unbecoming.'

In order to apply contraposition, we must put the premise into the form of **A**, thus 'All unbecoming things are unpleasing things,' the contrapositive of which is 'All not unpleasing things are not unbecoming things,' which having a double negative in each term equals 'All pleasing things are becoming.' We can regain the original premise by applying contraposition to this last result.

28. Convert, and give some immediate inferences from the following: 'Nothing is harmless that is mistaken for a virtue.'

The predicate of this proposition is clearly 'harmless,' and 'that is mistaken for a virtue' is a relative clause describing the subject. The proposition is then 'Nothing mistaken for a virtue is harmless' (**E**), converted simply into another proposition in **E**, 'Nothing harmless is mistaken for a virtue.'

Applying obversion to the original proposition we get

'All that is mistaken for a virtue is not-harmless,' or 'is harmful.' By immediate inference by complex conception, we infer 'All foolish conduct mistaken for virtue is harmful foolish conduct.' (Concerning inference by complex conception, see Thomson's *Outline*, § 88, and *Elementary Lessons*, p. 87.)

29. Because every Prime Minister is a man, can we infer that every good Prime Minister is a good man?

The process of immediate inference by added determinants, as described by Dr. Thomson, allows us to join an adjective or determining mark to both terms of an affirmative judgment, narrowing both terms, but to the same extent. Of course, however, it must be the same determining mark in each case, and if an adjective be ambiguous it is not logically the same adjective in its several meanings. Now good applied to a Prime Minister means that he is an able, active, upright minister, but probably very different from men who are good in other ranks of life. A good man means one who is good in the ordinary business and domestic relations of life. Thus the inference is erroneous. (See *Elementary Lessons*, p. 86.) It will afterwards be shown that when the proposition is fully expressed no such failure of inference can occur. Strictly speaking the premise is

Prime Minister = Prime Minister, Man;

and it follows inevitably that

Good, Prime Minister = Good, Prime Minister, Man.

30. Euler employed two overlapping circles to represent a particular proposition. Can you raise any objection to the accuracy of such a diagram?

Such circles have been employed in a great number of logical works. In my *Elementary Lessons* (p. 75) the particular proposition 'some metals are not brittle,' is represented by the following figure :—

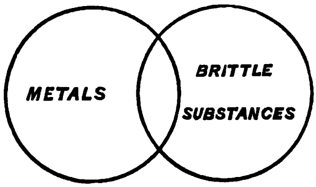


FIG. 1.

It does not seem to have been sufficiently noticed that though such a diagram correctly shows the exclusion of a part of the class metals from any part, that is all parts, of the class brittle substance, it indicates at the same time that another part of the class metals is included among brittle substances. Thus the diagram corresponds to the two propositions **I** and **O**, instead of showing either apart from the other. Now, it has been fully explained that **O** is consistent with the truth of **E**; so that when we say 'some metals are not brittle,' it may be that no metals are brittle, which is contradictory to **I**, 'some metals are brittle.' The diagram should not prejudice this question, and it would therefore be best to remove the part of the circle bounding metals which falls within the circle of brittle sunstances, or else to have a broken line, as in Fig. 2.

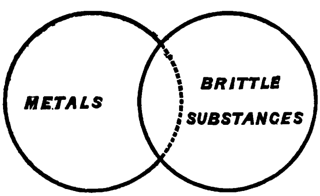


FIG. 2.

In the same way the proposition **I**, for instance, 'Some crystals are opaque,' would be represented by a broken circle included within a complete circle, in the manner shown either in Fig. 3 or Fig. 4.

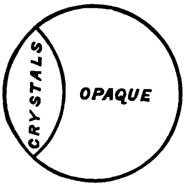


FIG. 3.

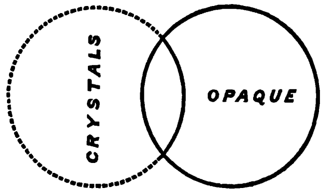


FIG. 4.

31. What is the logical force of the following sentence from Sidgwick's *Methods of Ethics:* 'A materialist will naturally be a determinist; a determinist need not be a materialist'?

Taking 'naturally' to give a universal force to the first proposition, it becomes 'All materialists are determinists.' The second proposition informs us that 'a determinist need not be a materialist,' that is to say, at the least, 'some determinists are not materialists.' This proposition is the sub-contrary of the converse of the first, and is the contradictory of 'all determinists are materialists.' The second proposition, then, prevents us from supposing materialists and determinists to be two coextensive terms. We learn that there are persons called materialists who are all found among determinists; hence some called determinists are found among materialists; other determinists, however, are not among materialists, and as to those who are not determinists, they cannot be materialists. The first proposition would be technically described as **A**, and the second as **O**, the contradictory of the inverse of the first.

32. 'All equilateral triangles are equiangular.' May we thence infer that triangles having unequal angles have unequal sides, and *vice versâ?*

The proposition asserts that all equal-sided triangles have equal angles; hence we may by contraposition infer that triangles which have not equal angles cannot have equal sides. But as the proposition stands, we are not justified in reading it *reciprocally* (see p. 32), and inferring that triangles which have not equal sides have not equal angles. This is true as a matter of geometrical science, but it is the contrapositive of another proposition, namely, the inverse 'all equiangular triangles are equilateral,' the truth of which must be separately proved.

33. Can we ever convert a proposition of the form 'all *A*s are *B*s' into one of the form 'all *B*s are *A*s'?

Certainly we cannot infer that all *B*s are *A*s because all *A*s are *B*s. As a general rule the predicate of the convertend *B* will be a wider term than the subject *A*, so that the inverse could not be inferred. Professor Henrici (*Elementary Geometry*, Congruent Figures, p. 14), for instance, describes space as a *three-way-spread*, but we cannot convert simply, and say that every three-way-spread is space. It nevertheless happens not uncommonly that the original proposition is really intended to mean 'all *A*s are all *B*s,' which can then be simply converted. Thus if space be defined as a *three-way-spread of points*, we can convert into every three-way-spread of points is space. Such definitions are of the form of proposition afterwards described by the symbol **U** (chapter xviii.), and considerable care is requisite in discriminating between the propositions **A** and **U.** J. S. Mill has pointed to the simple conversion of a universal affirmative proposition as a very common form of error (*System of Logic*, book v., chapter vi., section 2). It cannot be too often repeated that the reciprocal and inverse propositions as described on p. 32, cannot be inferred from an original of the form **A**.

34. In what cases does predication involve real existence? Show that in some processes of conversion assumptions as to the existence of classes in nature have to be made; and illustrate by examining whether any such assumptions, and if so what, are involved in the inference that if all S is P, therefore some not-S is not P.

The above question must have been asked under some misapprehension. The inferences of formal logic have nothing whatever to do with real existence; that is, occurrence under the conditions of time and space. No doubt, if all S is P, it follows that, in order to avoid logical contradiction, some not-S must be admitted to be not P. For instance, if 'All heathen gods are described in writings more than 1000 years old,' it follows that 'Some things which are not heathen gods are not described in writings more than 1000 years old.' This involves no assertion of real existence, nor could such an inference ever be drawn, unless, indeed, the original proposition itself asserted existence in time and space. This subject is pursued in a subsequent chapter.

CHAPTER VI

EXERCISES ON PROPOSITIONS AND IMMEDIATE INFERENCE

1. EXAMINE the following pairs of propositions, and decide which pairs contain consistent propositions, such that if the first of the pair be true the second may be true; and *vice versâ*, if the second be true, the first may be true. Give the technical name of the logical relation, if any, between the two propositions of each pair.

(1) { Some metals are useful.
All metals are useful.

(2) { No metals are useless.
Some useful things are not metals.

(3) { Some useless things are metals.
All useful things are metals.

(4) { Some metals are useful.
No metals are useless.

(5) { All metals are useful.
Some useless things are not metals.

2. Draw all the immediate inferences you can from the proposition 'Knowledge is power.'

3. Give the converse of the contrapositive of the proposition 'All organic substances contain carbon.'

4. Give all the logical opposites of *intuta quæ indecora*, 'Unsafe are all things unbecoming.'

5. What information about the term 'solid body' can we derive from the proposition, 'No bodies which are not solids are crystals'?

6. 'Only British subjects are native born Englishmen.' What precisely does this proposition tell us about the four terms—

British subject.
Not-British-subject.
Native born Englishmen.
Not-native-born-Englishmen?

7. Describe the logical relation between each of the four following propositions, and each of the other three:—

(1) All substances possess gravity which are material.
(2) No substances which possess gravity are immaterial.
(3) Some substances which are immaterial do not possess gravity.
(4) Some substances which do not possess gravity are immaterial.

8. State the nature and technical name of the logical process by which we get each of the following propositions from the preceding one:—

All men are mortal.
No men are immortal.
No immortals are men.
None but mortals are men.
All not-mortals are not men.
No men are not-mortals.
All men are mortals.

9. What are the subaltern propositions corresponding to the following universal propositions?—

(1) Every effect follows from a cause.
(2) No one is admitted without payment.
(3) All trespassers will be prosecuted with the utmost rigour of the law.
(4) Nemo me impune lacessit.

10. Give the obverse, converse, inverse, and reciprocal of each of the following propositions :—

(1) All mammalia are vertebrate animals.
(2) Sir Rowland Hill is dead.
(3) That which is a merit in an author is often a fault in a statesman.
(4) Whatever is necessary exists.
(5) In veritate victoria.

11. Give the contrary, contradictory, subaltern, converse, obverse, inverse, contrapositive, and reciprocal propositions corresponding to each of the following propositions :—

(1) All B.A.'s of the University of London have passed three examinations.
(2) All men are sometimes thoughtless.
(3) Uneasy lies the head which wears a crown.
(4) The whole is greater than any of its parts.
(5) None but solid bodies are crystals.
(6) He who has been bitten by a serpent is afraid of a rope.
(7) He who tries to say that which has never been said before him, will probably say that which will never be repeated after him.

12. Give as many equivalent logical expressions as you can for the propositions—

(1) If the treasury was not full, the tax-gatherers were to blame.

(2) Through any three points not in a straight line a circle may be described.

(3) It is false to say that only the virtuous prosper in life. [R.]

13. What logical relations are there between the following propositions?—

(1) All elementary substances are undecomposable.

(2) There are no compounds which are not decomposable.

(3) Some compounds are not decomposable.

(4) No undecomposable substances are compounds. [E.]

14. From the proposition 'Perfect happiness is impossible' can we infer that 'Imperfect happiness is possible'?

15. Is it the same thing to affirm the falsity of the proposition 'Some birds are predatory,' and to affirm the truth of the proposition 'Some birds are not predatory'?

16. Explain the statement that in the case of subcontrary propositions, truth may follow from falseness, but falseness cannot follow from truth.

17. Give in succession (1) the obverse, (2) the converse, (3) the subaltern, (4) the contrary, (5) the contradictory, (6) the contrapositive of the proposition 'All wise acts are honest acts.'

18. Concerning the same proposition answer the following questions:—

(1) How is its converse related to its subaltern?

(2) How is its converse related to the converse of its subaltern?

(3) How is its subaltern related to its contradictory?

[BAGOT.]

19. What is the converse of the contrary of the contradictory of the proposition 'Some crystals are cubes'? How is it related to the original proposition?

20. What is the converse of the converse of 'No men are ten feet high'?

21. Name the logical process by which we pass from each of the following propositions to the succeeding one:—

(1) All metals are elements.
(2) No metals are non-elements.
(3) No non-elements are metals.
(4) All non-elements are not metals.
(5) All metals are elements.
(6) Some elements are metals.
(7) Some metals are elements.

22. (1) 'None but a logical author is a truly scientific author.' Taking this proposition as a premise, examine the following propositions, and decide which of them can be inferred from the premise.

(2) A truly scientific author is no author who is not logical.
(3) Some truly scientific authors are not any authors who are not logical.
(4) A not truly scientific author is not a logical author.
(5) Those who are not truly scientific authors cannot be logical.
(6) All logical authors are truly scientific.
(7) No truly scientific author is an illogical author.
(8) All not illogical authors are truly scientific.
(9) No illogical author is a truly scientific author.
(10) No one is a truly scientific author who is not a logical author.
(11) Some logical authors are not truly scientific authors.

Give, as far as possible, the technical name of the logical relation between each of the above propositions and each other.

23. 'Some small sects are said to have no discreditable members, because they do not receive such, and extrude all who begin to verge upon the character.' Point out how this statement illustrates logical conversion.

24. Can we logically infer that because heat expands bodies, therefore cold contracts them?

25. Does it follow that because every city contains a cathedral, therefore the creation of a city involves the creation of a cathedral, or the creation of a cathedral involves the creation of a city?

26. All English Dukes are members of the House of Lords. Does it follow by immediate inference by complex conception that the creation of an English Duke is the creation of a member of the House of Lords?

27. Give *every possible* converse of the following propositions—

(1) Two straight lines cannot enclose space.
(2) All trade-winds depend on heat.
(3) Some students do not fail in anything. [M.]

28. Give the logical opposites, converse and contrapositive, of Euclid's (so-called) twelfth axiom—

If a straight line meet two straight lines, so as to make the interior angles on the same side of it taken together less than two right angles, those straight lines being continually produced shall at length meet upon that side on which are the angles which are less than two right angles.

29. How is the above proposition related to this other:— If a straight line fall upon two parallel straight lines, it

makes the two interior angles upon the same side together equal to two right angles? [R.]

30. From 'Some members of Parliament are all the ministers' (*Elementary Lessons*, p. 325, No. 3 [4]), can we infer that 'some place-seeking prejudiced and incapable members of Parliament are all the place-seeking prejudiced and incapable ministers'?

31. Is it perfectly logical to argue that *because* two sub-contrary propositions may both be true at the same time, *therefore* their contradictories, which are contrary to each other, may both be false?

32. Is it perfectly logical to argue thus?—If contrary propositions are both false, their respective contradictories, which are sub-contraries to each other, are both true. Now as this result is possible, it is therefore possible that the contraries may both be false.

33. What is the logical relation, if any, between the two assertions in Proverbs, chap. xi. 1, 'A false balance is abomination to the Lord: but a just weight is his delight'?

34. Examine the verses of Proverbs, chaps. x. to xv., and assign the relation between the two opposed assertions which make nearly all the verses.

35. What is the nature of the step from 'anger is a short madness' to 'madness is a long passion'? [R.]

36. 'The angles at the base of an isosceles triangle are equal.' What can be inferred from this proposition by obversion, conversion, and contraposition, without any appeal to geometrical proof?

37. From the assertion 'The improbable is not impossible,' what can we learn, if anything, about (1) the possible, (2) the probable, (3) the not-improbable, (4) the impossible, (5) the not-impossible?

38. How would a logician express the relations between the following statements of four interlocutors?—

(1) None but traitors would do so base a deed.
(2) And not all traitors.
(3) Some would.
(4) No; not even traitors.

[College Moral Science Examination, Cambridge.]

39. What difficulties or absurdities do you meet in converting the following propositions?—

(1) Some books are dictionaries.
(2) No triangle has one side equal to the sum of the other two.
(3) Every one is the best judge of his own interests.
(4) A few men are both scientific discoverers and men of business.
(5) Whatever is, is right.
(6) Some men are wise in their own conceit.
(7) 'The eye sees not itself,
But by reflection, by some other things.'

CHAPTER VII

DEFINITION AND DIVISION

1. ALMOST all text-books of Deductive Logic give rules for judging of the correctness of definitions, and for dividing up notions into subaltern genera and species. On attempting, however, to treat these parts of logic in the manner of this work, it has come home to me very strongly that they are beyond the sphere of Formal Logic, and involve the matter of thought. In form there is nothing peculiar to a definition; in fact the very same proposition may be a definition to one person and a theorem to another. It is open to us for instance to define the number 9 as $9 = 3 \times 3$; or, $9 = 8 + 1$; or $9 = 7 + 2$, etc.; but having selected at will any one of these equations as a definition, the other equations follow as theorems. The perplexity in which the theory of parallel lines is involved partly arises from the fact that there is choice of definitions, some mathematicians choosing one way and some the other. It is quite apparent, too, that the same proposition may afford different knowledge to different people. For instance, 'John Herschel was the only son of William Herschel' would serve as a definition of John Herschel to any one who knew only William Herschel, and of William Herschel to one who only knew John. To one who knew both it might be a

theorem. Similar remarks might be made concerning the distinction between ampliative and explicative propositions.

2. These in addition to other considerations convince me that any attempt to treat definition as a part of Formal Logic must be theoretically unsound and practically unsatisfactory. The case is somewhat similar with Logical Division, which, so far as it belongs to Formal Logic, can be nothing more than that method of Dichotomous Division fully developed in the later chapters on Equational Logic. Anything more than this must involve material knowledge, and should be treated in a different work and in a different manner. On these grounds I have decided not to attempt any explication of Definition and Division here, but to confine this chapter to a collection of questions, such as are to be commonly found in examination papers. The student may be referred for the current doctrines to the *Elementary Lessons*, Nos. XII. and XIII.; Fowler's *Deductive Logic*, Chapters VII. and VIII.; Duncan's *Logic*, Chapter VI. etc.

3. Examine the following definitions—

(1) Conversion is the changing of terms in a proposition.

(2) Opposed propositions are those which differ in quantity and quality.

(3) Contradictory opposition is the opposition of contradictories. [R.]

4. Define any of the following terms, notions, or classes of objects—

Gravitation	Franchise	Communism
Consistency	Imagination	Honour
Library	Honesty	Club
Vegetable	Revenge	Dictionary

Diet	Syllogism	Conservative
Hypochondriac	Racehorse	University
Merit	Success	Specie.

5. Criticise the following definitions—

(1) A square is a four-sided figure of which the sides are all equal and the angles all right angles.

(2) A member of the solar system is anything over which the sun has continued influence.

(3) La vie est le contraire de la mort.

(4) A lemma is a proposition which is only used as subservient to the proof of another proposition.

(5) An archdeacon is one who exercises archidiaconal functions.

(6) Life is the definite combination of heterogeneous changes, both simultaneous and successive, in correspondence with external coexistences and sequences.

(7) A gentleman is a man having no visible means of subsistence. [ORTON.]

(8) Equal bodies are those whereof every one can fill the place of every other. [HOBBES.]

6. Examine the definitions—

(1) Tin is a metal lighter than gold.

(2) Vice is the opposite of virtue.

(3) Paper is a substance made of rags.

(4) Cheese is a caseous preparation of milk.

(5) Rust is the red desquamation of old iron.

(6) A transcendental function is any function which is not an algebraic function.

(7) A right-angled triangle is a triangle containing one right angle, and of which the containing sides are or are not equal.

(8) An organ is any part of an animal or plant appropriated to a distinct function.

(9) A man is a self-knowing animal.

(10) Knowledge is that on which somebody else can be examined. [ROLLESTON.]

(11) An animal is a sentient organised being.

(12) A triangle is a three-sided figure having its angles together equal to two right angles.

(13) A man is one who may be the Prince of Transylvania. [HOBBES.]

7. In what respects are the following definitions, or some of them, defective?

(1) Logic is a guide to correct reasoning.

(2) Logic is the art of expressing thoughts in correct language.

(3) Logic is a mental science.

(4) Logic is the science of the regulative laws of human thought.

8. Does the eleventh chapter of the Hebrews, or any part of it, contain a correct logical definition of Faith?

9. Give examples of indefinable words, and explain why words may be indefinable.

10. Give the Proximate Genera for the following species—

Man	Plant	Monarchy
Triangle	Hound	Science.

11. Define by genus and differentia the following terms; and name a proprium and an accident in each case:—

Island	Parallelogram
Bank	Bill of exchange
Dictionary	Tree

12. What are the genus, species, difference, property, and accident of *Examination?* [D.]

13. Distinguish specific attribute, property, and accident, using the term *Circle* as an example. [B.]

14. Is it possible to define the terms gold, coal, legal nuisance, civilisation, Cleopatra's needle?

15. Define the term *boat*, and then point out how many of the following things the definition includes:—Bark, ferry-boat, floating fire-engine, pontoon, wherry, canoe.

16. Classify the following objects under one or other of the heads, *cash*, *bills*, *specie*:—

Cheque, promissory note, shilling, money, token-coin, bank-note, I.O.U., paper-money, sovereign, Scotch bank-note. (See *Money and the Mechanism of Exchange* (*International Scientific Series*), p. 248. Section on the Definition of Money.)

17. Distinguish Logical from Physical Division and Definition. [O.]

18. Can anything admit of more than one definition? [O.]

19. Distinguish precisely between the definition and the description of a class.

20. Explain the difficulties which arise concerning the definition of parallel straight lines, and criticise the following suggested definitions:—

(1) Lines which are in every part equidistant.
(2) Lines of similar direction.
(3) Lines which being in the same plane and indefinitely prolonged never meet.

21. Examine the following definitions:—

(1) Man is a bundle of habits.
(2) Law is common sense.
(3) Reverence is the feeling which accompanies the recognition of superiority or worth in others.
(4) Hunger is the product of man's reflection on the necessity of food. [P.]

22. Which of the following are logical divisions, and which are not?—

(1) Man into civilised and uncivilised.
(2) The world into Asia, America, Europe, Africa, Australia.
(3) Grammar into syntax and prosody.
(4) War into civil and aggressive.
(5) Syllogisms into those which are logical and illogical.
(6) Sequences of phenomena into casual and causal.
(7) Energy into potential and visible.
(8) Geometrical figures into plane and tri-dimensional.
(9) Allegiance is either natural and perpetual, or local and temporary.

23. Divide the term Inference, so as to include the various species usually discussed by logicians. [E.]

24. The following were the classes of persons which were in 1868 qualified to vote in one or other of the United States of America:—Male citizen, male inhabitant, every man, white male citizen, white freeman, male person, white male adult, free white male citizen, free white man.

Form a scheme of logical division which shall have a place for each of the above classes.

25. Divide *logically*—University, colour, chair, science,

religion, species, art, church, undergraduate, virtue, statesman. [O.]

26. Form a scheme of division of sciences to include the species — Deductive, experimental, concrete, descriptive, rational, abstract, inductive, explanatory, empirical.

27. Apply the rules of logical division to the following instances, correcting what is wrong, and supplying what is deficient :—

(1) Discursive thought may be divided into the Term, Judgment and Syllogism.
(2) Notions are Concrete, Singular, and Universal.
(3) Propositions are Affirmative, Negative, and Universal.

28. To what extent are the rules of division, usually given in logical treatises, repudiated by the classifications adopted in the Natural Sciences? [L.]

29. When is a division inadequate? When indistinct? When a cross division? And when not arranged according to proximate parts? [MORELL.]

30. Give an accurate scheme of logical division in which the following things shall find places :—Name; Part of Speech; Term; *Vox logica;* Verb; Noun, Adjective; Syncategorematic term; Word.

CHAPTER VIII

SYLLOGISM

1. MEDIATE Inference, or Syllogism, forms the principal part of Deductive Logic, and offers a wide scope for useful exercises. I give, in the first place, a brief epitome of the syllogistic rules and forms; I then exemplify them abundantly by question and answer; lastly, I supply chapters full of the largest and most varied collection of syllogistic questions and problems which has ever been published. Some of the more perplexing questions, involving the distinction of formal and material falsity of syllogisms and their premises, are treated apart in the succeeding chapter (xii.).

RULES OF THE SYLLOGISM

(1) *Every syllogism has three and only three terms.*

These terms are called the major term, the minor term, and the middle term.

(2) *Every syllogism contains three and only three propositions.*

These propositions are called respectively the major premise, the minor premise, and the conclusion.

(3) *The middle term must be distributed once at least.*

(4) *No term must be distributed in the conclusion which was not distributed in one of the premises.*

(5) *From negative premises nothing can be inferred.*

(6) *If one premise be negative, the conclusion must be negative;* and vice versâ, *to prove a negative conclusion one of the premises must be negative.*

From the above rules may be deduced two subordinate rules, which it will nevertheless be convenient to state at once.

(7) *From two particular premises no conclusion can be drawn.*

(8) *If one premise be particular, the conclusion must be particular.*

FIGURES OF THE SYLLOGISM

S = minor term. M = middle term. P = major term.

	First Figure.	Second Figure.	Third Figure.	Fourth Figure.
Major Premise.	M ... P	P ... M	M ... P	P ... M
Minor Premise.	S ... M	S ... M	M ... S	M ... S
Conclusion.	S ... P	S ... P	S ... P	S ... P

MOODS OF THE SYLLOGISM

The following is a compact table of the valid moods of the syllogism, the numerals showing the figures in which each group of propositions makes a valid syllogism :—

AAA	AAI	AEE	AII	AOO
1.	3. 4.	2. 4.	1. 3.	2.
EAE	EAO	EIO	IAI	OAO
1. 2.	3. 4.	1. 2. 3. 4.	3. 4.	3.

MNEMONIC VERSES

Barbara, *Celarent*, *Darii*, *Ferioque*, prioris ;
Cesare, *Camestres*, *Festino*, *Baroko*, secundae ;

Tertia, *Darapti*, *Disamis*, *Datisi*, *Felapton*,
Bokardo, *Ferison*, habet; Quarta insuper addit,
Bramantip, *Camenes*, *Dimaris*, *Fesapo*, *Fresison*.

Certain letters in the above lines indicate the way in which the moods of the second, third, and fourth figures may be *reduced* to the first figure, as follows—

s directs you to convert simply the proposition denoted by the preceding vowel.

p directs you to convert the proposition *per accidens*, or by limitation.

m, for *muta*, directs you to transpose the premises.

k denotes that the mood can only be reduced *per impossibile.*

The initial consonant of each mood in the three last figures corresponds with the initial of the mood of the first figure to which it is reducible.

QUESTIONS AND ANSWERS

2. State the figure and mood to which the subjoined argument belongs:—

Iron is not a compound substance; for iron is a metal, and no metals are compounds.

The conjunction 'for' shows that the proposition preceding is the conclusion—a universal negative. The term 'metal' must be the middle term, because it does not appear in the conclusion. The major term being 'compound substance,' the major premise must be 'no metals are compound substances,' (**E**) and the other premise 'iron is a metal' must be the minor. The latter is a universal affirmative; for though no mark of quantity is prefixed to 'iron,' it states a chemical truth concerning iron in general, and may fairly be interpreted universally (**A**). The argument belongs to the mood **E A E** in the first figure, or Celarent, thus :—

> **E** No metals are compound substances.
> **A** (all) Iron is a metal.
> **E** Iron is not a compound substance.

3. Examine the following argument; throw it into a syllogistic form, and bring out the figure and mood :—

   It cannot be true that all repression is mischievous, if government is repressive and yet is sometimes beneficial. [B.]

The conclusion is stated in the form of a denial of the universal affirmative 'all repression is mischievous'; hence the contradictory of this, or 'some repression is not mischievous' is the real conclusion. The middle term is 'government,' which does not appear in the conclusion. In looking for the major term, we do not find 'mischievous' n the premises, but only its opposite term 'beneficial.' We must assume, then, that we are intended to take 'beneficial' as equivalent to 'not-mischievous,' otherwise there would be a fallacy of four terms. To be brief, then, the syllogism takes this form—

> Some government is not mischievous.
> All government is repressive (or repression).

Therefore, 'some repression is not mischievous.' It is a valid syllogism in the third figure, and mood **O A O**, or Bokardo.

4. In what figures is the mood **A E E** valid?

In the first figure we have

All M is P.
No S is M.
No S is P.

The negative conclusion distributes the major term P, which is undistributed in the major premise; hence Illicit Process of the Major Term.

In the second figure we have

All P is M.
No S is M.
No S is P.

The major term is now properly distributed in the major premise, and the middle term being also distributed once, in the minor negative premise, the syllogism is valid in Camestres.

The reader may show that in the third figure we have again Illicit Process of the Major, and in the fourth figure a valid syllogism Camenes.

5. What rules of the syllogism are broken by arguments in the pseudo-moods, **O A E**, and **O I E**?

The answer cannot be better given than in the words of Solly (*Syllabus of Logic*, p. 86). In the mood **O A E** the predicate is distributed in the major premise, and the subject in the minor premise, and both subject and predicate in the conclusion. Hence it follows that either some term must be distributed in the conclusion which was not distributed in the premises, or else the middle term cannot be distributed in either premise. We cannot, therefore, determine at once which form the fallacy will take, but may be quite certain that there must be either an illicit process of major or minor, or else an undistributed middle.

Again, in the mood **OIE**, both subject and predicate are distributed in the conclusion, whereas no term is distributed in the minor premise, and it therefore follows that there must be an illicit process of the minor. It is also evident that the middle term cannot be distributed in the minor premise, and that if it is distributed in the major premise the major term must be undistributed, and consequently there must be a fallacy either of undistributed middle or illicit major.

6. None but whites are civilised; the ancient Germans were whites: therefore they were civilised. [W.]

This appears at first sight to be in Barbara, the terms standing apparently in the order of the first figure. But the major premise does not assert that all whites are civilised; it only asserts that *none but* whites are so, and this is equivalent to the contrapositive of the proposition

All civilised are whites.

Joining to this the minor premise

The ancient Germans were whites,

we see that the argument is in the second figure, with two affirmative premises, so that the middle term is undistributed in both cases, producing Fallacy of Undistributed Middle. There is also a difference of *tense* between the two premises which might perhaps invalidate an argument; but this point need not be further noticed here.

7. None but civilised people are whites; the Gauls were whites: therefore they were civilised. [W.]

At first sight this seems to be in the second figure, and invalid; but converting the major premise by contraposition, as in the last example, we get a valid syllogism in Barbara—thus, 'All whites are civilised; the Gauls were whites, etc.'

8. All books of literature are subject to error; and they are all of man's invention; hence all things of man's invention are subject to error. [H.]

This may seem at the first reading to be correct reasoning, especially as the conclusion is materially true; but there is fallacy of Illicit Process of the Minor Term. The argument is in the pseudo-mood **AAA** of the third figure, and the conclusion should be '*some* things of man's invention are subject to error.'

9. He who is content with what he has is truly rich; a covetous man is not content with what he has; no covetous man, therefore, is truly rich.

The middle term is 'content with what he has,' and since this term appears as subject of the major premise and predicate of the minor, the argument is in the first figure in the pseudo-mood **AEE**. There is Illicit Process of the Major Term, because the conclusion **E** distributes its predicate and the major premise **A** does not.

The conclusion may be true in matter but does not follow from the premises. We could only make the argument good by taking as major premise, 'All the truly rich are content with what they have.' This would give a valid syllogism in Camestres, but the original premise, if converted, only yields 'Some truly rich are content with what they have.'

10. Protection from punishment is plainly due to the innocent; therefore, as you maintain that this person ought not to be punished, it appears that you are convinced of his innocence. [W.]

The above is equivalent to—

The innocent are not to be punished;
This person is not to be punished;
Therefore, this person is innocent.

Put in this form there is an obvious fallacy of Negative Premises; but we can also express the premises in an affirmative form as follows:—

The innocent ought to be exempt from punishment;
This person ought to be exempt from punishment.

In this case it is apparent that the middle term 'ought to be exempt from punishment' is undistributed in both the premises, against Rule 3 of the syllogism.

11. 'He that is of God heareth my words: ye therefore hear them not, because ye are not of God.' [W.]

In the usual order:—

He that is of God heareth my words;
Ye are not of God;
∴ Ye do not hear my words.

The propositions are **AEE** in the first figure, and involve the Fallacy of Illicit Process of the Major Term. 'Hear my words' is distributed as the predicate of the negative conclusion, but is undistributed as the predicate of the affirmative major premise. The argument would become valid, however, if we were allowed to quantify this predicate universally, and assume the meaning to be

He that is of God = he who heareth my words.

12. Any books conveying important truths without error deserve attention; but as such books are few, it is plain that few books do deserve attention.

Carefully distinguish the truth and fallacy in this argument.

A good example suggested by Whately's No. 13; it may be thus put:—

Any books conveying, etc., deserve attention;
Few books do convey, etc.;
∴ Few books do deserve atttention.

This is in the first figure, and, if we interpret the conclusion to mean that '*A few* books do deserve attention,' that is to say, affirmatively only, without implying that the rest do not, it is valid in the mood Darii. But usually (*Elementary Lessons*, p. 67), we interpret *few* negatively; indeed, in the example itself this is the plain meaning, 'such books are few,' implying that all but this few do not convey important truths without error. This makes the minor premise into **O**, 'Most books do not, etc.,' and the argument consisting of **AOO** in the first figure is a case of Illicit Process of the Major Term.

13. That man is independent of the caprices of Fortune, who places his chief happiness in moral and intellectual excellence: A true philosopher is independent of the caprices of Fortune: therefore a true philosopher is one who places his chief happiness in moral and intellectual excellence. [W.]

A case of the fallacy of Undistributed Middle, the middle term 'independent of the caprices of Fortune,' being predicated affirmatively both of 'one who places his chief happiness, etc.,' and of 'a true philosopher.' The pseudo-mood is, therefore, **AAA** in the second figure. The fallacy is none the better because the conclusion may be considered true in matter. If the premise had begun 'Only that man is independent, etc.,' we might have put the argument into a valid syllogism, Barbara.

14. It is an intensely cold climate that is sufficient to freeze quicksilver; and as the climate of Siberia does this it is intensely cold.

At the first glance this looks like a case of Undistributed Middle; but we soon see that the major premise is really 'Any climate sufficiently cold to freeze quicksilver is an intensely cold climate.' The argument is thus valid in Barbara.

15. No one who lives with another on terms of confidence is justified, on any pretence, in killing him: Brutus lived on terms of confidence with Cæsar: therefore he was not justified, on the pretence he pleaded, in killing him. [W.]

This is valid in Barbara, the major term being 'justified on any pretence, etc.,' the middle term, 'one who lives, etc.,' and the minor term, 'Brutus.'

The conclusion, however, is obviously weakened, or is less general than it might have been. We might conclude that Brutus was not justified in killing Cæsar *on any pretence.* It is only inferred that he was not justified in killing him on the pretence he pleaded, which is of course included in 'any pretence.'

16. Inquire into the validity of the following argument: Whatever substance is properly called by the name Coal consists of a carbonaceous substance found in the earth; now, as this specimen consists of a carbonaceous substance, and was found in the earth, therefore it is properly called Coal. [L.]

The above argument is evidently a case of Undistributed Middle, because we infer that this specimen is properly called Coal on the ground of two universal affirmative propositions, both of which have the same predicate consisting of a carbonaceous substance found in the earth.' The pseudo-mood then is **AAA** in the second figure.

Though entirely failing in a demonstrative point of view, it is another question whether the specimen may not be believed to be coal, on analogical or inductive inference.

17. Give any remarks which occur to you concerning the following: 'Nerve power does not seem to be identical with electricity; for it is found that when a nerve is tightly compressed nervous action does not go on, but electricity can nevertheless pass.'

Implies the following syllogism :—

All tightly compressed nerves do not convey nervous action ;
All tightly compressed nerves do convey electricity ;
Therefore, some things which convey electricity do not convey nervous action.

The propositions are **AAI** in the third figure, *i.e.* the syllogism is valid in Darapti. It is matter of further inference that, because electricity is conducted by some things which do not convey nervous action, therefore these actions are not identical.

18. 'With some of them God was not well pleased; for they were overthrown in the wilderness.' [W.]

An enthymeme of the first order, the major premise being omitted. The order of statement is that by some logicians called analytical, the conclusion being put first, and the minor premise adduced as a reason or proof. The major premise, assumed to be obvious, is to the effect that 'All who are overthrown in the wilderness are among those with whom God is not well pleased.' More fully stated, indeed, the assumption might be that all who suffer from any signal calamity are some of those with whom God is not well pleased. To be overthrown in the wilderness is to suffer from a signal calamity. This view makes a sorites which the reader can put in order.

19. If the major term of a syllogism be the predicate of the major premise, what do we know about the minor premise? [L.]

In answering syllogistic questions of this sort, great attention must be given to throwing the reasoning into the briefest and clearest form. Such questions, thus treated, afford capital exercises in reasoning. The above question may be answered thus:—

If the major premise is affirmative its predicate, the major term, is undistributed and must likewise be undistributed in the conclusion; in this case the conclusion, and consequently the minor premise, must be affirmative. If the major premise be negative, then the minor premise must be affirmative, in order to avoid negative premises; thus in any case the minor premise is affirmative. Or still more briefly thus:—

The minor must be affirmative, for if negative then the major would have to be affirmative, which would involve Illicit Process of the Major.

20. Prove that **O** cannot be a premise in the first or fourth figure; and that it cannot be the major in the second figure, or the minor in the third. [M.]

If either of the premises be **O**, the conclusion must be negative, so that its predicate the major term will be distributed. But as **O** distributes only its predicate, and the other premise, which must of course be affirmative and universal, only distributes its subject, the syllogistic conditions are much restricted. Thus **OA** as premises in the first figure give an Undistributed Middle, the middle term being subject of **O** and predicate of **A**. The premises **AO** give Illicit Process of the Major Term, the predicate of **A**, the major term, being undistributed. In the second figure **O** cannot be the major, because its subject would then be the major term, and undistributed. In the third figure it cannot be the minor, because the major term would then be predicate of **A**, the major premise, and thus Illicit Process of the Major would again arise. Finally, in the fourth figure **OA** will give Illicit Major, and **AO**, Undistributed Middle.

21. If it be known concerning a syllogism in the Aristotelian system, that the middle term is distributed in both premises, what can we infer as to the conclusion?

The syllogism cannot be in the second figure, because the middle term, being the predicate in both premises, these would both have to be negative, against Rule 5. In the first figure the minor premise would have to be negative, in order to distribute its predicate, the middle term; but a negative minor in the first figure gives Illicit Process of the Major Term. In the third figure, however, the middle term being subject of both premises will be twice distributed if these be both universal, which happens in the moods Darapti and Felapton. In the fourth figure the middle term is predicate of the major and subject of the minor; we must, therefore, have a negative major and a universal affirmative minor, which happens in the mood Fesapo. We find, then, that a doubly distributed middle term can prove only a particular conclusion, **I** or **O**, and these only in the third and fourth figures.

22. Take an apparent syllogism subject to the fallacy of negative premises, and inquire whether you can correct the reasoning by converting one or both of the premises into the affirmative form.

[India Civil Service, July 1879.]

Take premises in the first figure—

No Y is Z;
No X is Y.

Obvert the major premise (see p. 42), and we have—

All Y is not-Z;
No X is Y.

The premises would give no conclusion, the pseudo-mood **AEE** in the first figure involving Illicit Process of the Major. Obverting the minor, we have—

No Y is Z;
All X is not-Y.

There are now four terms, and therefore no common middle term at all. The reader may easily work out other examples. (See *Principles of Science*, p. 62; first edition, vol. i. p. 75.)

23. Prove that the third figure must have an affirmative minor premise, and a particular conclusion.

In the third figure the major term is predicate of the major premise. Now, if the minor premise be negative, the conclusion will be negative (Rule 6), and distribute its predicate the major term; but the major premise must be affirmative in order to avoid negative premises. Thus, there will arise Illicit Process of the Major Term. It follows, by *reductio ad absurdum*, that the minor premise cannot be negative and must be affirmative.

Again the predicate of the minor premise is the minor term, and, the premise being affirmative, this term will be undistributed, giving a particular conclusion.

24. Show that if the conclusion of a syllogism be a universal proposition, the middle term can be but once distributed in the premises.

Questions of this sort can be most briefly answered by counting the available number of distributed terms in the premises. Thus, if the conclusion be a universal affirmative proposition, we need one distributed term for its subject. But, as the premises must both be affirmative, they contain at most two distributed terms, namely their subjects. Hence there is only one place in which the middle term can be distributed. On the other hand, if the universal conclusion be negative, both major and minor terms require to be distributed in the premises; but, as one premise only can be negative, we cannot possibly have more than three terms distributed, the subject and predicate of the negative premise, and the subject of the affirmative one. Two being required for the major and minor terms, there remains only one distributed place for the middle term, which was to be proved.

Observe that this result is the contrapositive of that proved under Question 21 (p. 83).

25. Given the six rules of the syllogism, and the rule that two particular premises prove nothing, show that if one premise be particular the conclusion must be particular.

This may be demonstrated by the following ingenious reasoning of De Morgan (*Formal Logic*, p. 14).

'If two propositions, *P* and *Q*, together prove a third, R, it is plain that *P* and the denial of *R* prove the denial of *Q*. For *P* and *Q* cannot be true together without *R*.

Now, if possible, let *P* (a particular) and *Q* (a universal) prove *R* (a universal). Then *P* (particular) and the denial of *R* (particular) prove the denial of *Q*. But two particulars can prove nothing.'

26. Show that the proposition **O** is seldom admissible as a minor premise.

When **O** is the minor premise the conclusion must be negative by Rule 6, and will therefore distribute its predicate, the major term. As we must not have two negative premises by Rule 5, the major premise must be affirmative, and will not distribute its predicate. Hence the major term must be the subject of the major premise. Now, since the middle term becomes the undistributed predicate of the major premise, it must be the predicate of the minor premise, in order that it may be once distributed. Thus we conclude that **O** can be the minor premise only in the second figure, giving the mood Baroko.

27. Show that a universal negative proposition (**E**) is highly efficient as a major premise. [P.]

Since **E** has both its terms distributed, either of them may serve as the major term, which, the conclusion being negative, must be distributed. The other term will then serve to distribute the middle term once. The minor premise may therefore be chosen at will, provided that it be affirmative in order to avoid negative premises. There is no restriction of figure, and accordingly we find valid moods with **E** as major premise in all of the four figures; in fact, no less than eight of the nineteen recognised moods begin with **E**.

28. Name the weakened moods of the syllogism. In what figure can there be no weakened mood, and why? Do any of the nineteen moods commonly recognised give a weaker conclusion than the premises would warrant?

By a *weakened* mood is meant one which gives a particular conclusion when a universal conclusion might have been drawn. The information obtained from the premises is thus 'weakened.' This can, of course, happen only when the conclusion of the stronger mood is universal. Hence, in the third figure, which gives only particular conclusions, there can be no weakened mood. In the other figures each mood which has a universal conclusion will have a corresponding weakened mood with conclusion of the same quality. Thus Barbara gives a mood **AAI**; Celarent, **EAO**; Cesare, **EAO**; Camestres, **AEO**; in the fourth figure only Camenes admits of a weakened form, **AEO**. Thus the weakened moods are five in number.

Bramantip of the fourth figure is the single mood alluded to in the latter part of the question.

Considering that it is impossible to employ conversion by limitation without weakening the logical force of the premise, it is too bad of the Aristotelian logicians to slight the weakened moods of the syllogism as they have usually done.

29. Can we under any circumstances infer a relation between X and Z from the premises—

Some Ys are Xs;

Some Ys are Zs?

[India Civil Service, July 1879.]

Not if 'some *Y*s' bear the sense attributed to the expression in Logic. The indefinite adjective of quantity *some* is so indefinite, that it must never be interpreted twice over with the same meaning. But if the *some Y* in the one premise were intended by the arguer to be '*the same some Y*' as in the other premise, the term would practically become a distributed one, and the premises might give a valid conclusion in the mood Darapti. Dr. Thomson has remarked (*Laws of Thought*, § 77, p. 132), that 'the word (some) appears to be employed in the two senses of "Some or other," and "Some certain," in common language.' Observe, however, that it is in the former purely indefinite sense that logicians have always used the word, so that 'some *Y*' must not be identified with 'some *Y*.'

30. Is the following argumenta valid syllogism? That which has no parts cannot perish by the dissolution of its parts; the soul has no parts; therefore, the soul cannot perish by the dissolution of its parts.

This example is quoted from the *Port Royal Logic*, Part III. Chap. ix., Example 6. It is there remarked that several persons advance such syllogisms in order to show the inaccuracy of the unconditional rule (5) that 'nothing can be inferred from negative premises.' Without remembering what was said in the *Art of Thinking*, I made the same objection in the *Principles of Science*, p. 63 (first ed. vol. i. p. 76), and I must still hold that in its bare statement the syllogistic rule is actually falsified. But it must, no doubt, be allowed that if the premises are to be treated as both negative, then there are four terms; the middle term is broken up into two terms, 'that which has not parts,' and 'that which has parts'; the soul is denied to be the latter; the former is that of which it is asserted negatively that it cannot perish, etc. It comes simply to this, that the syllogistic rules are to be interpreted as a whole, and in making the above example conform to the first rule (see p. 71) we make it conform also to the fifth rule.

Professor Croom Robertson has criticised my treatment of this subject (*Mind*, 1876, p. 218, *note*), urging that 'There are four terms in the example, and thus no syllogism, if the premises are taken as negative propositions; while the minor premise is an affirmative proposition, if the terms are made of the requisite number three.' No doubt Professor Robertson is substantially right, but it may be noticed that my words were so cautious as hardly to commit me to an erroneous statement. I now find that the point has been treated by many logicians in addition to those of *Port Royal*, as for instance, Burgersdicius; De Morgan, *Formal Logic*, p. 139, Art. 3; Bain, *Deductive Logic*, p. 164; Devey, *Logic, or the Science of Inference*, 1854, p. 129; *Essai sur la Logique*, 1763, p. 106.

31. In reference to the syllogism, Mr. Jevons urges that it sometimes yields a conclusion that is open to misinterpretation, as in the example—

Potassium is a metal;
Potassium floats on water;
Therefore, Some metal floats on water.

Examine this criticism carefully.

[Moral Science Tripos, Cambridge, Dec. 1876.]

I said in the *Principles of Science* (pp. 59-60; first ed.

vol. i. pp. 71-2), that my inference, namely, 'Potassium metal = potassium floating on water,' is of a more exact character than the Aristotelian result 'Some metal floats on water.' The 'some' after all is only here an indefinite name for 'potassium,' and unless we constantly bear in mind that 'some' means in logic 'one, and it may be more or all,' the reasoner is apt to confuse 'some' with the plural 'several.' This view of the matter was criticised by Professor Croom Robertson (*Mind*, 1876, p. 219).

32. What is the nature of the argument, if any, in the apparent enthymeme, 'The field is neglected because the soil is poor'?

This may, of course, be an argument in Barbara, thus—

Every field of poor soil is neglected;
This is a field of poor soil;
∴ This field is neglected.

But the statement may also mean that the soil being poor is the *reason or cause* why the owner neglects it; in this case, it is not an argument but a causal relation. The student, therefore, must always look out for ambiguities in the conjunction 'for,' 'because,' etc., which may certainly bear one of two if not of more senses. The relation between premises and conclusion has nothing whatever to do with the relation between cause and effect.

33. Explain—'It is scarcely ever possible decidedly to affirm that any argument involves a bad syllogism; but this detracts nothing from the value of the syllogistic rules.' [R.]

Scarcely any one in ordinary writing or discourse states a syllogism in full form; it is always presumed that the hearer or reader is enough of a logician to supply what is wanting. Now the missing premise may generally be supplied in such a way as to make a good syllogism formally speaking, that is to say, so as to avoid any breach of the syllogistic rules. It is another matter whether the new premise is materially true. The value of the syllogistic rules is, then, that they enable us to assign the premises which would be requisite to support the conclusion put forward. They thus oblige the arguer to define the nature of his assumptions, or else to yield up his conclusion.

34. How shall we reduce the following syllogism to the first figure?—

All men are liable to err;
None who are liable to err should refuse advice;
None who should refuse advice are men.

This argument is in the absurd fourth figure, in the mood Camenes. In this name the letter *m* directs us to transpose the premises, and the final *s* directs us to convert the conclusion simply; making these changes, we obtain the same argument in the more natural form of Celarent, thus—

None who are liable to err should refuse advice;
All men are liable to err;
No men should refuse advice.

35. How shall we reduce the following syllogism to the first figure?—

All birds are vertebrates;
Some winged animals are not vertebrates;
Some winged animals are not birds.

The premises are **AO** in the second figure, and the conclusion being **O**, the argument is a valid syllogism in Baroko.

The letter *k* directs us to employ the *Reductio ad impossibile*, as explained in the *Elementary Lessons*, p. 149. Or we may convert the major by contraposition, getting—

All not-vertebrates are not birds;
Some winged animals are not-vertebrates;
Therefore, Some winged animals are not birds.

Taking the negative term not-vertebrates as the middle term, this is valid in Barbara.

36. Can we reduce the mood Camestres *per impossibile?*

Taking the symbolic example—

All *X*s are *Y*s;
No *Z*s are *Y*s;
Therefore, No *Z*s are *X*s,

and assuming for sake of argument that the conclusion is false, the contradictory 'some *Z*s are *X*s' will be true, which put as minor premise with the original as major, gives the valid syllogism in Darii—

All *X*s are *Y*s;
Some *Z*s are *X*s;
Therefore, Some *Z*s are *Y*s.

But this conclusion is the contradictory of the original minor premise 'no *Z*s are *Y*s,' so that we cannot contradict the conclusion of Camestres without producing a syllogism in Darii to contradict one of our original premises. Thus we prove the conclusion of Camestres indirectly by a mood of the first figure. It will be found on trial that all the moods of the imperfect figures may be similarly proved indirectly by one or other of the moods of the first or so-called *perfect figure.*

CHAPTER IX

QUESTIONS AND EXERCISES ON THE SYLLOGISM

1. ASSIGN the moods of the following valid syllogisms, pointing out in succession—

(*a*) The conclusion;
(*b*) The middle term;
(*c*) The major term, and the major premise containing it;
(*d*) The minor term, and the minor premise containing it;
(*e*) The quantities and qualities of the three propositions;
(*f*) Their symbols;
(*g*) The order in which they should be technically placed;
(*h*) The figure of the syllogism;
(*i*) The mood, and its mnemonic name.

(1) No birds are viviparous;
All feathered animals are birds;
No feathered animals are viviparous.

(2) Robinson is plain spoken; for he is a Yorkshire man, and all Yorkshire men are plain spoken.

(3) Birds are not viviparous animals;
Bats are viviparous animals;
Bats, therefore, are not birds.

(4) Whatever investigates natural laws is a science;
Logic investigates natural laws;
Logic is a science.

(5) Quicksilver is liquid at ordinary temperatures;
Quicksilver is a metal;
Some metal, therefore, is liquid at ordinary temperatures.

(6) True fishes respire water containing air;
Whales do not respire water containing air;
Whales, therefore, are not true fishes.

2. Arrange the following valid syllogisms in the usual strict order of major premise, minor premise and conclusion. Name the figure and mood to which they belong. In examining syllogisms, always follow the directions of the first question.

(1) Iridium must be lustrous; for it is a metal, and all metals are lustrous.
(2) Some pleasures are not praiseworthy; hence some pleasures are not virtuous, for whatever is not praiseworthy is not virtuous.
(3) Epicureans do not hold that virtue is the chief good, but all true philosophers do hold that it is so; accordingly, epicureans are not true philosophers.
(4) Some towns in Lancashire are unhealthy, because they are badly drained, and such towns are all unhealthy.

3. Draw conclusions from the following pairs of premises, specifying the figure and mood employed—

(1) Every virtue is accompanied with discretion;
There is a zeal without discretion.

(2) Sodium is a metal;
Sodium is not a very dense substance.

(3) All lions are carnivorous animals;
No carnivorous animals are devoid of claws.

(4) { Combustion is chemical union;
Combustion is always accompanied by evolution of heat.

(5) { All boys in the third form learn algebra;
There are no boys in the third form under twelve years of age.

(6) { Nihil erat quod non tetigit:
Nihil quod tetigit non ornavit.

4. Examine the following arguments and point out which are valid syllogisms, naming the figure and mood as before; in the case of such as are pseudo-syllogisms, name the rule of the syllogism which is broken thereby, and give the technical name of the fallacy—

(1) All feathered animals are vertebrates;
No reptiles are feathered animals;
Some reptiles are not vertebrates.

(2) Some vertebrates are bipeds;
Some bipeds are birds;
Some birds are vertebrates.

(3) All vices are reprehensible;
Emulation is not reprehensible;
Emulation is not a vice.

(4) All vices are reprehensible;
Emulation is not a vice;
Emulation is not reprehensible. [L.]

(5) Some works of art are useful;
All works of man are works of art;
Therefore some works of man are useful. [L.]

(6) Iron is a metal;
All metals are soluble;
Iron is soluble.

(7) Aryans are destined to possess the world;
Chinese are not Aryans;
Chinese are not destined to possess the world.

(8) Only ten-pound householders have votes;
Smith is a ten-pound householder;
Smith has a vote.

5. What are the suppressed premises which are evidently presumed to exist by those who set forth the following imperfectly stated syllogisms? State figure and mood as usual.

(1) Blessed are the meek: for they shall inherit the earth.
(2) This iron is not malleable; for it is cast iron.
(3) Whosoever loveth wine shall not be trusted of any man; for he cannot keep a secret.
(4) Being born in Africa, he was naturally black.
(5) Some parallelograms are not regular plane figures, for they cannot be inscribed in a circle.
(6) Suffer little children to come unto me; for of them is the kingdom of Heaven.
(7) It is dangerous to tell people that the laws are not just; for they only obey laws because they think them just.
(8) The line *A B* is equal to the line *C D*; for they are both radii of the same circle.
(9) Whales are not true fishes, for they respire air; moreover they suckle their young.
(10) The Queen is at Windsor, for the royal standard is flying.
(11) The science of logic is very useful; it enables us to detect our adversaries' fallacies.
(12) He must be in York, for he is not in London.

(13) I shall not derive my opinions from books, for I have none. [Mansfield, H. of L., 1780.]

(14) The nation has a right to good government; therefore it may rebel against bad governors.

(15) The wise man has an infinity of pleasures; for virtue has its delights in the midst of the severities that attend it.

6. Point out which of the following pairs of premises will give a syllogistic conclusion, and name the obstacle which exists in other cases.

(1) No *A* is *B*; some *B* is not *C*.
(2) No *A* is *B*; some not *C* is *B*.
(3) All *B* is not *A*; some not *A* is *B*.
(4) Some not *A* is *B*; no *C* is *B*.
(5) All not *B* is *C*; some not *A* is *B*.
(6) All *A* is *B*; all not *C* is not *B*.
(7) All not *B* is not *C*; all not *A* is not *B*.
(8) All *A* is not *B*; no *B* is not *C*.
(9) All *C* is not *B*; no *A* is not *B*.

7. To what moods do the following belong?—

(1) 'All *B* is *A*; only *C* is *A*; therefore only *C* is *B*.'

(2) 'All *B* is *A*; nothing but *C* is *A*; therefore nothing but *C* is *B*.'

See Dante's *De Monarchia*, as translated by F. C. Church, and appended to the *Essay* on *Dante*, by the Rev. R. W. Church, 1878, p. 195. Many curious specimens of reasoning, sometimes pedantic, might be drawn from the *De Monarchia*.

8. Supply premises to prove or disprove the following conclusions—

(1) The loss of the *Captain* proves that turret-ships are not sea-worthy.

(2) The cottage-hospital system should be adopted.

(3) The Prussians are justified in refusing the rights of war to Garibaldi if they find him fighting against them. [E.]

(4) Private property should be respected in war.

(5) No woman ought to be admitted to the franchise. [O.]

(6) The law of libel requires to be amended. [O.]

(7) Capital punishment ought to be abolished. [O.]

(8) Royal parks ought not to be used for political meetings. [O.]

(9) Written examinations are not a safe test of merit.

(10) Written examinations are a safe test of merit. [E.]

(11) The Annuity-tax should be done away with. [E.]

(12) Any national system of education should be a secular system. [E.]

9. In how many different moods may the argument implied in the following question be stated? 'No one can maintain that all persecution is justifiable who admits that persecution is sometimes ineffective.'

How would the formal correctness of the reasoning be affected by reading 'deny' for 'maintain'? [C.]

10. What conclusions, and of what mood and figure, can be drawn from each pair of the following propositions?

(1) None but gentlemen are members of the club.

(2) Some members of the club are not officers.

(3) All officers are invited to dine.

(4) All members of the club are invited to dine. [C.]

11. Express the following reasonings in each of the four syllogistic figures.

(1) Some medicines should not be sold without registering the buyer's name, for they are poisons. [E.]

(2) No unwise man can be trusted; hence some speculative men are unworthy of trust, for they are unwise. [E.]

12. Can the following argument be stated in the form of a syllogism, and if so, what is the middle term?—

'The power of ridicule is a dangerous faculty, since it tempts its possessor to find fault unjustly, and to distress some for the gratification of others.'

13. If the proposition 'warmth is essential to growth' occurred as the premise of a syllogism, would you treat 'warmth' as a distributed or an undistributed term? [E.]

14. Show that the following single propositions may be regarded as enthymemes, that is, as equivalent to imperfectly expressed syllogisms:—

(1) Have thou nothing to do with that just man. [W.]

(2) If wishes were horses, beggars would ride.

(3) Large colonies are as detrimental to the power of a State, as overgrown limbs to the vigour of the human body.

(4) If I had read as much as my neighbours, I would have been as ignorant. [HOBBES.]

(5) All law is an abridgment of liberty and consequently of happiness.

(6) Thales being asked what was the most universally enjoyed of all things, answered—Hope; for they have it who have nothing else.

(7) I will give thee my daughter if thou canst touch heaven.

(8) If all the absurd theories of lawyers and divines were to vitiate the objects in which they are conversant, we should have no law and no religion left in the world. [BURKE.]

15. Distinguish between the causal, simply logical, or other, senses of the copulative conjunctions in the following—

(1) It will certainly rain, for the sky looks black.
(2) The people are happy because the government is good.
(3) This plant is not a rose; for it is monopetalous.
(4) The ancient Romans trusted their soothsayers, and must therefore have been frequently deceived.
(5) A favourable state of the exchanges will lead to importation of gold: this will cause a corresponding issue of bank-notes which will occasion an advance in prices; which again will check exportation and encourage importation, tending to turn the exchanges against us. [GILBART, 1851, p. 284.]

16. Form an example of a syllogism in which there are two prosyllogisms, one attached to the middle and the other to the minor term. [H.]

17. Prove that a valid sorites with n premises must have $n + 1$ terms, and is capable of giving $\frac{n(n-1)}{2}$ conclusions.

18. Can the following Shakspearean passage (*Hamlet*, Act v. Scene i.) be stated in the form of a sorites?

'Alexander died, Alexander was buried, Alexander returneth into dust; the dust is earth; of earth we make loam; and why of that loam, whereto he was converted, might they not stop a beer barrel?'

19. Throw the reasoning of the following passage into syllogistic form :

'Carbon, which is one of the main sources of the nourishment of plants, cannot be dissolved in water in its simple form, and cannot therefore be absorbed in that form by plants, since the cells absorb only dissolved substances. All the carbon found in plants must consequently have entered them in a form soluble in water, and this we find in carbonic acid, which consists of carbon and oxygen.' [A.]

20. Complete such of the following arguments as may be considered sound but incomplete syllogisms :—

(1) The people of the country are suffering from famine, and as you are one of the people of the country, you must be suffering from famine.

(2) Light cannot consist of material particles, for it does not possess momentum.

(3) Aristotle must have been a man of extraordinary industry; for he could not otherwise have produced so many works.

(4) Marcus Aurelius was both a good man and an Emperor; hence it follows that Emperors may be good men, and *vice versâ*.

(5) Nothing which is unattainable without labour is valuable; some knowledge is not attainable with labour, and is therefore valuable.

(6) All gasteropods are mollusks, and no vertebrate animals are mollusks; therefore no gasteropods are vertebrate.

(7) Suicide is not always to be condemned; for it is but voluntary death, and voluntary death has been gladly embraced by many great heroes.

CHAPTER X

TECHNICAL EXERCISES IN THE SYLLOGISM

1. PROVE, from the general rules of Syllogism, that when the major term is predicate in its premise, the minor premise must be affirmative.

2. Prove that, when the minor term is predicate in its premise, the conclusion cannot be a universal affirmative. [L.]

3. Prove that there must always be in the premises one distributed term more than in the conclusion.

4. Prove that the major premise of a syllogism, whose conclusion is negative, can never be a particular affirmative.

5. Prove that when the minor premise is universal negative, the conclusion (unless weakened) will be universal.

6. Prove that, if in the first figure we transpose the major premise and conclusion, we obtain a pseudo-mood.

7. In the third figure, if the conclusion be substituted for the major premise, what will the figure be? [BAGOT.]

8. Prove that no syllogism in the fourth figure can be correct which has a particular negative among its premises, or a universal affirmative for its conclusion. [L.]

9. If the major term be universal in the premises and particular in the conclusion, determine the mood and figure, it being understood that the conclusion is not a weakened one. [C.]

10. Why is it impossible to transform the mood **AEO** from the second figure into the first?

11. What figure must have a negative conclusion? Why must it?

12. What figure must have a particular conclusion? Why must it?

13. Why must the major premise of the fourth figure not be **O**?

14. Why must the minor premise of the fourth figure not be **O**?

15. If the minor premise of the first figure were not affirmative, what fallacy would be committed?

16. If the major premise of the first figure were **I**, what fallacy would be committed?

17. What kind of proposition does not occur in the premises of the first figure? Why does it not occur?

18. If the major premise of the second figure were particular, what fallacy would be committed?

19. What is remarkable about the conclusions of the third figure?

20. What kind of proposition cannot be proved by the fourth figure?

21. In what mood of the syllogism can a subaltern proposition be substituted for its subalternans (universal of same quality) as premise without affecting the conclusion?

22. If one premise of a syllogism be **O**, what must the conclusion be?

23. Prove that a universal affirmative proposition can form the conclusion of the first figure alone.

24. Why is **OAO** excluded from the first and second figures?

25. Why is it that the moods **EAO** and **EIO** are true in all the four figures?

26. It is said that **A** is the most difficult conclusion to establish by the syllogism, and the most easy to overthrow; **O**, on the contrary, is the most easy to establish, and the most difficult to overthrow. What is there in the moods of the syllogism to support this view?

27. 'When both premises of an apparent syllogism are negative, the real middle term is an external sphere, and is consequently undistributed' [SOLLY]. Explain the meaning of this statement.

28. If the minor premise of a syllogism be **O**, what is the figure and mood?

29. Prove that there cannot be more than four figures of the syllogism.

30. If one premise be **O**, what must the other be?

31. Show that in the fourth figure the conclusion may be either affirmative or negative, and, if negative, either universal or particular.

32. Given the major premise particular, or the minor premise negative, what must the other premise be? Why so? [P.]

33. If the minor term be predicate of minor, or major subject of major, the conclusion may not be **A**? [P.]

34. Prove that the combination of a particular major premise with a negative minor premise leads to no valid inference.

35. Prove that wherever there is a particular conclusion without a particular premise, something superfluous is invariably assumed in the premises.

36. Determine in what affirmative moods the middle term may be universal in the major premise, and particular in the minor.

37. Determine in what negative moods the same may occur. [P.]

38. Determine how many universal terms may be in the premises more than in the conclusion.

39. Determine how many particular terms may be in the premises more than in the conclusion.

40. Determine in what cases there may be in a syllogism an equal number of universal terms and of particular. [P.]

41. How do you reduce Camestres, Festino, Darapti, Fresison to the first figure?

42. Exemplify the reduction of Baroko and Bokardo by the process *per impossibile.*

43. Show that Cesare, Disamis, Camenes, and other moods can likewise be proved *per impossibile.* (See Karslake, 1851, vol. i. p. 81.)

44. Reduce Celarent to the fourth figure. To how many other figures can you reduce it?

45. Reduce Felapton to the second figure.

46. To what other moods, respectively, can you reduce Darii, Ferio, Barbara?

CHAPTER XI

CUNYNGHAME'S SYLLOGISTIC CARDS

1. IN this age of mechanical progress it may be a matter of surprise that no one has produced a syllogistic machine. About two centuries ago Pascal and Leibnitz invented true calculating machines, and Swift, incited perhaps by the accounts of these machines, described the professors of Laputa as in the possession of a thinking machine. About thirty years ago the late Alfred Smee, F.R.S., proposed the construction of a kind of mechanical dictionary, together with a contrivance for comparing the ideas defined in it. More recently I have constructed a machine which analyses the meaning of any propositions worked upon its keys, provided they do not involve more than four distinct terms. Yet the rules of the syllogism have never been put into a mechanical form. So much the worse for the rules of the syllogism.

2. Some approximation to a syllogistic machine has, however, been recently made by Mr. Henry Cunynghame, B.A. He has devised certain cards, which, if placed one upon the other, infallibly give a syllogistic mood when that is possible, and when it is not, indicate the absence of a conclusion. The contents of the cards, too, can be condensed into a hollow cylinder turning upon a solid cylinder, in such a way as to give all possible syllogistic

moods in one turn of the handle. This device, though hardly perhaps to be called a *syllogistic machine*, is probably the nearest approximation to such a machine which is possible. I am now enabled, by Mr. Cunynghame's kindness, to describe these ingenious and interesting devices for the first time.

3. The syllogistic cards consist of a set of eight larger and a set of eight smaller cards. Each larger card is $3\frac{1}{2}$ inches high by $2\frac{1}{2}$ inches broad, and bears, near to its upper edge, one of the eight possible propositions connecting *M*, the middle term, with *P*, the major term. There are, of course, four propositions of the forms **A**, **E**, **I**, **O**, in which *M* is the subject, and four more in which *P* is the subject. In the lower part of each larger card is written, in a certain position, each proposition between *S* and *P*, which can possibly be drawn from a syllogism having the proposition on the upper edge of the card for its major premise. Thus, under the major premise 'Some *M* is *P*,' we find 'Some *S* is *P*,' as the only conclusion that can be drawn from it; but the universal affirmative proposition, 'All *P* is *M*,' admits of any one of three possible conclusions, namely, 'No *S* is *P*,' 'Some *S* is *P*,' or 'Some *S* is not *P*.'

4. Each card of the lesser set is also $2\frac{1}{2}$ inches wide, but only 3 inches high, and bears on its upper edge one of the eight possible minor premises connecting *S*, the minor term, with *M*, the middle term. In the lower part of each smaller, or, as we may call it, *minor card* (with one exception), is cut one, or in some cases two rectangular openings, so adjusted that if any minor card be placed upon a larger or *major* card, so that the major and minor premises inscribed upon them be visible, one below the other, the conclusion, if any, belonging to those premises is seen through the opening in the minor card. If two conclusions, a universal proposition,

and its subaltern minor, are both possible, both will be seen. Thus, if we take the major 'Some P is M,' and the minor 'All M is S,' we see below the conclusion 'Some S is P,' the mood being Dimaris in the fourth figure. If we take the minor 'All S is M,' and place it upon the major 'No M is P,' we see below 'No S is P,' the correct conclusion in the mood Celarent, together with 'Some S is not P,' the corresponding weakened or subaltern conclusion. If the major be 'Some P is not M,' or the minor 'Some M is not S,' no conclusion can appear at all.

5. The cards are shown in complete detail on the next page. The principle on which they are constructed is that of excluding illogical conclusions. No conclusion is written on the major card but such as the premise at the top will warrant, and no conclusion is left uncovered by the minor card if it be unwarranted by the minor premise. The student can readily cut out cards according to the directions given above and the figures on the next page, but they must be cut exactly to scale. It will assist the construction if each major card be divided by a pencil mark into seven horizontal spaces, each half an inch high, each minor card being divided into six similar spaces.

6. The syllogistic cylinder is more compact and handy, but much less easy to describe and illustrate. The principle is exactly the same, but considerable ingenuity was needed to combine the cards together into cylinders in the best way. The order of the minor premises on the inferior or moving cylinder is as follows:—'All S is M,' 'All M is S,' 'Some S is M,' 'Some M is S,' 'No S is M,' 'No M is S,' 'Some S is not M,' 'Some M is not S.' The major premises are in like order, 'All P is M,' 'All M is P,' 'Some P is M,' etc.

CUNYNGHAME'S SYLLOGISTIC CARDS

Major Cards

All *M* is *P* All *S* is *P* Some *S* is *P*	No *M* is *P* No *S* is P Some *S* is not *P*	Some *M* is *P* Some *S* is *P*	Some *M* is not *P* Some *S* is not *P*
All *P* is *M* No *S* is *P* Some *S* is *P* Some *S* is not *P*	No *P* is *M* No *S* is *P* Some *S* is not *P*	Some *P* is *M* Some *S* is *P*	Some *P* is not *M*

Minor Cards

All *S* is *M*	No *S* is *M*	Some *S* is *M*	Some *S* is not *M*
All *M* is *S*	No *M* is *S*	Some *M* is *S*	Some *M* is not *S*

CHAPTER XII

FORMAL AND MATERIAL TRUTH AND FALSITY

1. THE rules of syllogistic inference teach us how, from certain premises assumed to be true, to draw other propositions which will be true under those assumptions. But if, instead of supposing the premises to be true, we regard them as materially false, various puzzling questions arise as to the conclusions which may properly be drawn. Such questions have not been adequately treated in any popular manual of logic; and as they lead to results of great practical importance, and at the same time furnish admirable exercises in the discrimination of good and bad reasoning, I propose to draw special attention to this subject in the following questions with answers.

2. Is it possible to draw a false conclusion from true premises?

It is possible, of course, to draw any conclusion from any premises, if we disregard the principles of logic and of common sense. But when we speak in a logical work of drawing a conclusion, we must be understood to mean drawing a conclusion logically, in accordance with the Laws of Thought and the Rules of the Syllogism. Now the nature of the logical relation between premises and conclusion is this, that if the premises are true the conclusion is true; truth is, as it were, carried from the premises into the conclusion; not the whole truth necessarily, but nothing except truth. The question above must, of course, be answered with a direct negative.

3. Is it possible to prove a true conclusion with false premises?

To prove a true conclusion, or to prove that a certain conclusion is true, must mean to establish its truth in the opinion of the persons concerned. 'To prove,' says Wesley, 1832, p. 90, 'is to adduce premises which establish the truth of some conclusion.' Now, the relation of premises and conclusion in a syllogism, as stated just above, is that if the premises are true the conclusion must be admitted to be likewise true. But, if certain persons regard the premises as false, they cannot possibly regard such premises as establishing or proving the truth of the conclusion. Solly, indeed, points out (1839, note, p. 9) 'the possibility of a true conclusion from false premises in every form of reasoning.' But this remark can only mean that the conclusion is *materially* true, or known to be true, on other grounds.

4. If the premises of a syllogism are false, does this make the reasoning false?

No. The reasoning is correct if the form of the premises and conclusion agree with that of any valid mood of the syllogism or other development of the Laws of Thought, wholly regardless of the material truth of any of the propositions *per se*. The most ridiculous proposition may make a good syllogism: for instance—

Every griffin has angles equal to two right angles;
Every triangle is a griffin;
Therefore, Every triangle has, etc.

This is, of course, a valid syllogism in Barbara, and if the premises were true the conclusion would be true; the premises being untrue, the truth of the conclusion is entirely unaffected by the reasoning.

5. (*a*) 'The most perfect logic will not serve a man who starts from a false premise.'
(*b*) 'I am enough of a logician to know that from false premises it is impossible to draw a true conclusion.'
Comment carefully upon the two foregoing extracts.

Both the above sentences have been written in perfect seriousness by men of intelligence, and they are fair specimens of the logic which would pass muster almost anywhere except in a book of logical exercises. In the first place, as regards (*a*), if a premise be materially false it cannot give a conclusion materially true; accidentally, indeed, it might do so by paralogism; but, as the logic is assumed to be 'most perfect,' we are dealing only with material truth and falsity. Nevertheless, the logic may serve the man well; for he can learn the truth or falsity of his propositions by observing their congruity with external facts. Now, if he has failed to learn in this way directly the falsity of his premise, his only chance is to draw logical conclusions from that premise, and then observe whether they are or are not materially verified. It is quite clear that if by correct logic we reach a conclusion materially false, then we must have started from premises which involved material error. This procedure represents, in fact, the real method of induction, as the *inverse process of deduction*, by which we learn all the more complicated truths of physical and moral science. (See *Principles of Science*, Chaps. XI. and XII.) The sentence (*a*), then, is true only on the supposition that a man, having adopted a false premise, will blindly accept all its false results, that is to say, will reason in a purely deductive manner.

The sentence (*b*) is erroneous, because, as we have fully learnt (p. 112), we can from false premises *draw* a true conclusion in good logical form. But there was doubtless confusion in the writer's mind between formal and material falseness, and had he said that by premises materially false it is impossible to establish the material truth of any conclusion, he would have been correct.

6. An apparent syllogism of the second figure being examined is found to break the rules of the syllogism, the middle term being undistributed. On further examination it is remarked that one of the premises is evidently false, and the other true. What can we infer from such circumstances concerning the truth or falsity of the conclusion? [C.]

As in the second figure the middle term is predicate in both the premises, the apparent syllogism can break the third rule of the syllogism, requiring that it shall be distributed once at least, only when both the premises are affirmative. The premises must therefore be **AA**, **AI**, **IA**, or **II**. In the first case, if the premises be

All *X*s are *Y*s,
All *Z*s are *Y*s,

and we assume the first one to be false, we obtain its contradictory as true; thus

Some *X*s are not *Y*s;
All *Z*s are *Y*s.

The conclusion must, by Rule 6, be negative, and there will be Illicit Process of the Major. Assuming the second premise to be false, we get

All *X*s are *Y*s;
Some *Z*s are not *Y*s.

Whence we may correctly infer in the mood Baroko,

Some *Z*s are not *X*s.

In the case of **AI**, we obviously cannot assume **A** to be false; but if **I** be false, we get

All *X*s are *Y*s;
No *Z*s are *Y*s;
Therefore, No *Z*s are *X*s.

In the premises **IA**, we cannot assume **A** to be false without Illicit Process of the Major Term; but if **I** be false, we have Cesare. Lastly, in **II** only the major can be taken as false, and its contradictory then gives us a syllogism in Festino. If the conclusion of the apparent or pseudo-syllogism in question does not correspond with what we thus obtain, the conclusion is logically false as compared with the new premises assumed.

7. If (1) it is false that whenever *X* is found *Y* is found with it, and (2) not less untrue that *X* is sometimes found without the accompaniment of *Z*, are you justified in denying that (3) whenever *Z* is found there also you may

be sure of finding *Y*? And however this may be, can you in the same circumstances judge anything about *Y* in terms of *Z*? [R.]

This excellent example of reasoning by contradictories can be easily solved by adhering to the simple rules of opposition, and gradually undoing the perplexities. The supposition that, whenever *X* is found *Y* is found with it, may be stated as the universal affirmative 'all *X*s are *Y*s'; but as this is false, its contradictory 'some *X*s are not *Y*s' is the true condition. That (2) *X* is sometimes found without the accompaniment of *Z*, would mean that 'some *X*s are not *Z*s'; but, being asserted to be untrue, the real condition is its contradictory 'All *X*s are *Z*s.' Thirdly, whenever *Z* is found, there also you may be sure of finding *Y*, means that 'all *Z*s are *Y*s'; but, if you deny this, you must assert that 'some *Z*s are not *Y*s.' Putting these propositions together, thus—

(1) Some *X*s are not *Y*s;
(2) All *X*s are *Z*s;
Hence, (3) Some *Z*s are not *Y*s;

we find that they make a valid syllogism in the third figure, and the mood Bokardo. The conclusion, being a particular negative, cannot be converted directly; we can only obtain by obversion and conversion 'some not-*Y*s are *X*s.' Thus we must, I presume, answer the last part of the problem negatively.

8. What is the precise meaning of the assertion that a proposition—say 'All grasses are edible'—is false?

The doctrine of the falsity of propositions is generally supposed to be defined with precision in the ancient formula of the square of opposition. If a universal affirmative proposition is false, its contradictory, the particular negative, is true, so that, in the case of the example given above, we infer, that 'some grasses are not edible.' Similarly, from the falsity of **E** we infer the truth of **I**, and *vice versâ*. But it does not seem to have occurred to logicians in general to inquire how far similar relations could be detected in the case of disjunctive and other more complicated kinds of propositions. Take, for instance, the assertion that 'All endogens are *all* parallel-leaved plants.' If this be false, what is true? Apparently that one or more endogens are not parallel-leaved plants, as, else that one or more parallel-leaved plants are not endogens. But it may also happen that no endogen is a parallel-leaved plant at all. There are three alternatives, and the simple falsity of the original does not show which of the possible contradictories is true.

But the question arises whether there is not confusion of ideas in the usual treatment of this ancient doctrine of opposition, and whether a contradictory of a proposition is not any proposition which involves the falsity of the original, but is not the sole condition of it. I apprehend that any assertion is false which is made without sufficient grounds. It is false to assert that the hidden side of the moon is covered with mountains, not because we can prove the contradictory, but because we know that the assertor must have made the assertion without evidence. If a person ignorant of mathematics were to assert that 'all involutes are transcendental curves,' he would be making a false assertion, because, whether they are so or not, he cannot know it. Professor F. W. Newman has correctly remarked that no one can really believe a proposition the terms of which he does not understand (*Lectures on Logic*, 1838, pp. 35, 36).

This is unquestionably true; for, if he does not know what things he is speaking about, he cannot possibly bring them to comparison in his mind. A witness who swears that a prisoner did a certain act when, as a matter of fact, he does not know whether the prisoner did it or not, swears falsely, independently of the question whether rebutting evidence can be brought to prove the perjury. It is reported that a man, who wished to be thought an acquaintance of Dr. Johnson, remarked to him in coming out of church, 'A good sermon to-day, Dr. Johnson.' 'That may be, sir,' replied the very much over-estimated doctor, 'but I'm not sure that you can know it.' This hits the point precisely.

It will be shown in a subsequent chapter that a proposition of moderate complexity has an almost unlimited number of contradictory propositions, which are more or less in conflict with the original. The truth of any one or more of these contradictories establishes the falsity of the original, but the falsity of the original does not establish the truth of any one or more of its contradictories, because there always remains the alternative that nothing is known concerning the relations of the terms. It may even happen that no relation at all exists between the terms. In this view of the matter, then, an assertion of the falsity of a proposition means its *simple deletion*. The contrariety is not between knowledge and knowledge, but between knowledge and ignorance.

It ought also to be remembered, in dealing with the doctrine of falsity, that the falsity of 'all *X*s are *Y*s' only implies that *one* or more *X*s are not *Y*s. Now in practice one or a few exceptions are often of no importance; there are in many cases singular exceptions which in a sense agree with, and in a sense falsify, a general proposition. Thus all points of a revolving sphere describe circles, excepting the two points at the poles. Other examples of singular exceptions will be found in the *Principles of Science*, Chapter XXIX. Professor Henrici points out (*Elementary Geometry*, 1879, p. 37) that a proposition must be considered to be true *in general*, if it be true in an infinite proportion of cases, and false only in a finite number of exceptions.

This subject of the truth and falsity of propositions as premises and conclusion may be pursued in Karslake's *Logic*, vol. i. p. 83; Whately, Book II. Chap. iii. § 2; *Aristotle*, *Prior Analytics*, Book II. Chaps. i.-iv.; *Port Royal Logic*, Part II. Chap. vii. Watts' *Logic*, Part II. Chap. ii. §§ 7 and 8.

Most of the scholastic logicians, such as Thomas Aquinas and Nicephorus Blemmidas, treat this subject elaborately.

9. 'Trust' (said Lord Mansfield to Sir A. Campbell) 'to your own good sense in forming your opinions; but beware of attempting to state the grounds of your judgments. The judgment will probably be right;—the argument will infallibly be wrong.'
Explain this phenomenon, and show its logical significance. [P.]

If you give reasons for a decision, implying that those reasons are sufficient, and are the reasons upon which you did make the decision, it is possible for critics subsequently to inquire whether such reasons logically support the conclusion derived from them. If they do not, the judge will be detected in a paralogism which there may be no means of explaining away. But, if no reasons be given, it will seldom be possible for critics to make any such detection. It is impossible, as a general rule, to publish in detail the law as well as the evidence upon which a law case is decided; and, even if it were published, it would generally be impossible to detect bad logic in a man who does not assign the precise points on which he relies, and the way in which he argues about a complex mass of details.

Although it may be, from his own point of view, convenient and discreet for a man to avoid giving reasons for any important public decision, if he can avoid it, yet it is an open question how far such means of escaping criticism is likely to increase the carefulness and impartiality of his judgments. There are many cases, including nearly all the verdicts given by juries on points of fact, where it would be highly undesirable to require any statement of reasons. Where the result depends upon oral testimony, the behaviour of witnesses, the estimation of degrees of probability and degrees of guilt, it is quite impossible to define and publish the real premises of the conclusion come to. We must trust to common sense and judicial tact. The same remarks may apply to various arbitrations, magisterial decisions, administrative acts, votes of members of deliberative bodies. But where the grounds of decision are precise and brief, so as to be capable of complete statement, it seems absurd to suppose that a judge will judge less well because he needs to disclose his argument. If he displays bad logic, where bad logic can be judged, he is clearly not fit to be a judge. Lord Mansfield's advice may possibly have been prudent and good when given to a man who was forced to act in novel circumstances, and in a distant colony (Jamaica), where his decisions would have more of the nature of administrative acts than law-building judgments. But the decisions of the High Court of Justice in England not only affect the parties in the cause, but shape the public law of a large part of the civilised world, and it is of course requisite that they should be guided by good logic.

CHAPTER XIII

EXERCISES REGARDING FORMAL AND MATERIAL TRUTH AND FALSITY

1. COMPARE the following syllogisms, or pseudo-syllogisms, both as regards their formal correctness, and as regards the material truth of their premises and conclusion; then explain how it is that a materially true conclusion is obtained in each case.

(1) All existing things are real things;
No abstract ideas are existing things;
∴ No abstract ideas are real things.

(2) No real things exist;
All abstract ideas are real things;
∴ No abstract ideas are real things.

(3) All real things are existing things;
No abstract ideas are existing things;
∴ No abstract ideas are real things.

2. If there be two syllogisms, of which we know that their major premises are subcontrary propositions, how may we determine the figure and mood of both? May their conclusions be both true in matter?

3. Prove by means of the syllogistic rules, that given the

truth of one premise and the conclusion of a valid syllogism, the knowledge thus in our possession is in no case sufficient to prove the truth of the other premise. [C.]

4. It is known concerning a supposed syllogism that it involves a fallacy of undistributed middle, and that one of the premises is false in matter; can we or can we not draw any conclusion under these circumstances?

5. Construct two syllogisms, such that the major premise of one shall be the subcontrary of the conclusion of the other, and such also that the conclusions of both shall be true in matter. Are these data sufficient to determine the figure?

6. If one premise be false in matter, and the syllogism correct in form, does it follow that the conclusion is false in matter?

7. Examine the doctrine 'that, if the conclusion of a syllogism be true, the premises may be either true or false; but that, if the conclusion be false, one or both of the premises must be false.'

8. Interpret the logical force of the following passage from Mr. Freeman's *Essay on the Holy Roman Empire*:— 'It may have been foolish to believe that the German King was necessarily Roman Emperor, and that the Roman Emperor was necessarily Lord of the world.'

9. Taking a syllogism of the third figure, and assuming one of the premises to be false, show whether or not, with the knowledge of its falsehood thus supposed to be in our possession, we can frame a new syllogism: if so, point out the figure and mood to which it will belong.

10. What do you mean by (1) Formal, (2) Material

truth, as applying (*a*) to a single proposition, (*b*) to a syllogism?

11. Give a careful answer to the miscellaneous example, No. 88, in *Elementary Lessons in Logic*, p. 322.

12. Is the following extract sense or nonsense, logically correct or incorrect? 'We may doubt whether the ancient method of reduction can prove the validity of any syllogistic mood; for, as from false premises we can illogically obtain a true conclusion, the *reductio ad impossibile* has doubts cast upon its validity as a method of proof.'

13. What is the precise meaning of the assertion that it is false to say that Castro cannot be proved not to be Orton?

14. If *P* asserts that oxygen, hydrogen, and nitrogen cannot be liquefied, and *Q* denies the assertion, what precisely must *Q* be understood to mean?

15. Analyse all that is implied in the assertion of the falsity of each of the following propositions:—

(1) Roger Bacon was a giant.
(2) Descartes died before Newton was born.
(3) Bare assertion is not necessarily the naked truth.
(4) All kinds of grass except one or two species are not poisonous.

16. Let *X*, *Y*, *Z*, *P*, *Q*, *R*, be six propositions: given

(*a*) Of *X*, *Y*, *Z*, one and only one is true;
(*b*) Of *P*, *Q*, *R*, one and only one is true;
(*c*) If *X* is true, *P* is true;
(*d*) If *Y* is true, *Q* is true;
(*e*) If *Z* is true, *R* is true;

Prove syllogistically, that

(*f*) If X is false, P is false;
(*g*) If Y is false, Q is false;
(*h*) If Z is false, R is false. [c.]

17. How do you meet the following difficulties?—

(1) True premises may by false reasoning give a correct conclusion; because, in a train of reasoning there may be two errors, and one error may neutralise the other.

(2) Since truth applies only to propositions, and a term *per se* is incapable of truth, it follows that a term must *per se* always be false, because everything must be either true or false.

CHAPTER XIV

PROPOSITIONS AND SYLLOGISMS IN INTENSION

1. To any one desirous of acquiring a thorough command of logical science, nothing is so important as a careful study of the intensive or comprehensive meaning of terms, propositions, and syllogisms. This indeed is not an easy task, as is shown by the fact that some great logicians who have written upon the subject, especially Sir W. Hamilton, have fallen into grave errors, or at the best fatal ambiguities of expression. Most of the common text-books, again, either ignore the subject altogether, or else treat it in a manner quite disproportioned to its difficulty and importance. The following questions and answers touch some of the more obscure points of the matter, but the student is assumed to have read the fifth of the *Elementary Lessons in Logic*, or else to have studied the subject in one or more of the following books: *Port Royal Logic*, Part I., Chapters v. to vii., Spencer Baynes' Translation, 1861, pp. 45-55 (this work was the first to draw attention to the subject in modern times); Watts' *Logic*, Part I., Chapter vi., §§ 9 and 10; Levi Hedge, articles 34 to 38; Thomson's *Outline of the Necessary Laws of Thought*, § 52; Spalding, 1857, §§ 30 to 33; Walker's *Commentary on Murray's Compendium*, Chapter II.; Bowen's *Treatise on Logic*, Chapter IV.

The most elaborate treatment of the subject is found in

Peirce, *Proceedings of the American Academy of Arts and Sciences*, 1867, vol. vii. pp. 416-432. If the student read Hamilton's *Lectures on Logic*, he must carefully observe what is said about Hamilton in this chapter.

2. State the proposition 'Men are mortals' in the intensive form.

This proposition, as it stands, is clearly *extensive*, and asserts that all individual men will be found among the things called mortals. When asked to turn such a proposition into the intensive form, students make all kinds of blunders, saying, for instance, that 'All the qualities connoted by the term man are connoted by the term mortal,' or 'All the properties of men are properties of mortal.' This is certainly not the case, because men, in addition to being mortal, are rational, are vertebrate, are erect, etc. Again, a student would say, 'The attributes of man connote the attributes of mortality.' This means nothing, the verb connote being wrongly used. To say, again, that 'All which possess the properties of man, possess the properties of mortal,' is to leave the proposition just as it was before, 'All which possess the properties of man,' being simply men.

In passing from the extensive to the intensive mode of thought, there must be a complete inversion of the relation of the terms. As men are a part of mortals, so the qualities of *mortals* are a part of the qualities of men. If we like we may use a different kind of copula; but, in that case, much care is necessary to avoid error. The following are different modes of expressing correctly the same truth, the first of each pair being an extensive and the second the corresponding intensive form of assertion.

All men are included among mortals;
All qualities of mortals are included among the qualities of men.

Mortals include men;
Properties of men include the properties of mortals.

Man is a species of mortal;
The genus mortal is in the species man.

Men are part of mortals;
Mortality is part of humanity.

3. Can we exhibit particular and negative propositions in the intensive form?

This question has not, I think, been much investigated by logicians, and the remarks to be found in the works of Hamilton and most other logicians apply only to the universal affirmative proposition. Taking the particular affirmative, 'Some crystals are opaque,' it asserts that 'One or more crystals are among opaque things.' It follows, no doubt, that the quality 'opaqueness' is among the qualities of one or more crystals, namely, *the particular crystals* referred to in the extensive proposition. Thus **I** may be treated intensively much as **A** is treated.

Taking the negative proposition, 'No iron bars are transparent,' we cannot infer that 'No properties of transparent objects are properties of iron bars.' This inference would be quite false; for, there may be many properties, such as gravity, inertia, indestructibility, extension, etc., which are possessed alike by transparent objects and iron bars. All we can infer is that 'Not all the properties of transparent things are in iron bars,' or, 'Some of the properties of transparent things are not in iron bars.' Entire separation in extension involves only partial separation in intension, or an extensive assertion in **E** gives an intensive assertion in **O**. We may also change **E** into **A**, getting 'All iron bars are non-transparent objects,' which of course gives the intensive form, 'The quality of non-transparency is among the qualities of iron bars.'

We may in a somewhat similar way treat the particular negative, say 'Some crystals are not symmetrical.' We cannot infer that 'All the common properties of symmetrical things are absent from some crystals,' but only *some* of those properties.

4. How shall we state the following syllogism in the intensive form?—

All crystals are solids;
All topazes are crystals;
Therefore, All topazes are solids.

We must transmute each of the three propositions into the intensive form, and transpose the premises, thus—

All qualities of crystals are qualities of topazes;
All qualities of solids are qualities of crystals;
Therefore, All qualities of solids are qualities of topazes.

The syllogism is turned, as it were, completely inside out.

5. Is Hamilton correct in stating the following as a valid syllogism?—

S comprehends *M*;
M does not comprehend *P*;
Therefore, *S* does not comprehend *P*.

Professor Francis Bowen (*Logic*, p. 237) has pointed out that, as the statement stands (see Hamilton's *Lectures*, vol.

iii. pp. 315, 316, American Edition, p. 223), it is simply illogical. It involves a fallacy of Illicit Process of the Major Term; in short, *S* may comprehend *P* through other means besides *M*. But Professor Bowen thinks that Hamilton's error lay in the choice of language, which was such as no one would understand it as Hamilton really intended it. The matter, however, is too important to be passed over in this way, and I proceed to notice other places in which Hamilton has treated of intensive syllogisms.

In his sixteenth Lecture on Logic (vol. iii. p. 295), we find the following :—

'An Extensive Syllogism.	An Intensive Syllogism.
B is *A*	*C* is *B*
C is *B*	*B* is *A*
C is *A*	*C* is *A*
All man is mortal;	Caius is a man;
But Caius is a man;	But all man is mortal;
Therefore, Caius is mortal.	Therefore, Caius is mortal.'

Between the syllogisms as thus stated there is no difference whatever, except the transposition of premises, a mere difference in the order of writing which is immaterial to the point in question. It is true that Hamilton goes on to explain as follows :—

'In these examples, you are aware, from what has previously been said, that the copula in the two different quantities is precisely of a counter meaning; in the quantity of extension, signifying *contained under;* in the quantity of comprehension, signifying *contains in it*.'

Afterwards the example of a concrete syllogism before given, is thus fully stated in the intensive form (p. 296).

'The Major term Caius contains in it the Middle term man;
But the Middle term man contains in it the Minor term mortal;
Therefore, the Major term Caius contains in it the Minor term mortal.'

To say the least, this is a very clumsy and misleading mode of explanation; for after all, the point of the matter is left untouched, namely, that it is individual things which are *contained under* in the extensive sense, and *qualities*, or attributes, which are contained in the other sense. Is it not absurd to say that Caius contains man, without explaining that *man* is here taken intensively? A thing does not contain any of its qualities in the same way that the class man, extensively regarded, contains one of its members or significates, namely, Caius. Nor is the matter much mended by referring back to the previous explanation (p. 274), where Hamilton illustrates the relations, saying, 'Thus the proposition,—*God is merciful*, viewed in the one quantity, signifies *God is contained under merciful*, that is, the notion *God* is contained under the notion *merciful;* viewed as in the other, means,—*God comprehends merciful*, that is, the notion *God comprehends in it* the notion *merciful*.' This is again all wrong, unless we interpret *notion* and *containing* in a totally different sense in the second as compared with the first statement. Even if Hamilton understood the matter correctly himself, he ought to have stated it unequivocally, and not to have left the reader to put the matter right by careful interpretation of the same words in two different senses. His subsequent exposition of the sorites is even worse; for he gives the identically same premises twice over in different order, and asserts, without other explanation, that one is in comprehension and the other in extension. Suppose it were stated in a police court that 'Brown struck Robinson,' and then 'Robinson struck Brown.' Should we not be surprised to learn subsequently that the verb 'struck' was used in a psychological sense in one case and a physical one in the other, the real meaning being that 'Brown struck Robinson as a very disagreeable fellow,' and then 'Robinson struck Brown on the head'? Yet this would not be worse than for Hamilton to state a syllogism or sorites twice over, with an unimportant change of order, and then assume that the reader takes one statement to be in extension and the other in intension.

I am obliged, therefore, to coincide with the opinion of De Morgan that Hamilton really missed the point of the question, in short, did not understand what he was writing about. The whole matter is put in the clearest light by the following few lines from De Morgan's *Syllabus*, pp. 62-63 :—

'The logicians who have *recently* introduced the distinction of extension and comprehension, have altogether missed this opposition of the quantities, and have imagined that the quantities remain the same. Thus, according to Sir W. Hamilton, "All *X* is some *Y*" is a proposition of comprehension, but "Some *Y* is all *X*" is a proposition of extension. In this the logicians have abandoned both Aristotle and the Laws of Thought from which he drew the few clear words of his dictum : "the genus is said to be part of the species ; but in another point of view (ἄλλως) the species is part of the genus." All *animal* is in *man*, notion in notion : *all man* is in *animal*, class in class. In the first, all the notion *animal* part of the notion *man* ; in the second, all the class *man* part of the class *animal*. Here is the opposition of the quantities.'

The same view is more fully stated in De Morgan's *Third Memoir on the Syllogism* (pp. 17–19 ; *Camb. Phil. Trans.*

1858, pp. 188-9). On the whole, I conclude that Hamilton's treatment of the subject is so doubtful and confusing that it had better not be studied in an elementary course of reading.

De Morgan, in the paper just referred to, gives some remarks about the history of the doctrine of intension and extension, and speaks of Hamilton as a logician who has 'recently contended for the revival, or rather the full introduction, of the distinction of extension and comprehension.' He correctly names the *Port Royal Logic* as being the first modern work to insist on the distinction, though the use made of it is 'not very extensive.' But he names only one other work, the *Institutiones Philosophicæ* of J. Bouvier (3rd Ed. Mans, 1830), as describing this distinction. De Morgan's reading of modern logic was not extensive. Not to mention the familiar Watts' *Logic*, in which the doctrine is frequently dwelt upon (see Part I. Chap. III. Section 3; Chap. VI. Section 10, and elsewhere), I find the matter excellently explained in 1816 in the brief manual of the American logician, Levi Hedge (pp. 42-44). In Murray's *Manual*, formerly much used in Dublin and Glasgow, the subject is fully explained, and in the clearest possible manner, in the Commentary of John Walker on Chapter II. This is in fact one of the best pieces of logical exposition which I know. Walker remarked that he had treated the point fully, because he regarded it as absolutely necessary to the understanding of the subsequent pages, which were often puzzling to students not familiar with the distinction between the comprehension and extension of a term. With some regret I must hold, then, that the pretensions of Hamilton in this matter are mistaken and unfounded.

The whole subject of extension and comprehension or intension has been investigated with much care and profundity of thought by the American logician, Professor C. S. Peirce, in the memoir already referred to (see p. 127). This memoir should be studied by those who wish to acquire a thorough understanding of logical principles and relations.

6. It is asserted by some logicians that the predicate of a proposition must be interpreted in intension while the subject is regarded in extension. Give your opinion upon this point, and explain the bearing of the question upon recent logical controversies. [C.]

I should answer this question to the effect that a proposition being, conformably to the opinion of Condillac, necessarily of the nature of an equation, it is absurd to suppose that things can be equated to their own qualities or circumstances. A proposition in extension expresses the identity of a thing or class of things with the same thing or class under another designation. As De Tracy says (*Idéologie*, vol. iii. p. 529), 'Dans tout jugement, les deux idées comparées sont nécessairement égales en *extension*.' A proposition in intension expresses an identity between the attributes of the one member and those of the other. The subject may be pursued in my Essay on *Pure Logic, or the Logic of Quality apart from Quantity*, 1864, *passim*; in J. S. Mill's *Logic*, Book I. Chapter V.; and in Dr. Martineau's review of Samuel Bailey on the Theory of Reasoning, in his *Essays Philosophical and Theological*, 1869, vol. ii.

CHAPTER XV

QUESTIONS ON INTENSION

1. 'Christian,' 'animal,' 'Episcopalian,' 'organised,' 'man.' Arrange these terms (1) in the order of extension, beginning with the most extensive; and (2) in the order of comprehension, beginning with the most comprehensive. [L.]

2. Arrange the following in the same manner :—General, animal, composer for the pianoforte, Roman, historian of his own campaigns, conqueror of Gaul. (See De Morgan, *Third Memoir*, pp. 20, 21.)

3. Arrange in order of extension and intension such of the terms given in Question 1 of Chapter II. as are the names of subaltern, genera, and species, and can be arranged in a series.

4. Analyse the following terms in the counter quantities or wholes of extension and intension: *Man*, *government*, *law*, *triangle*, *vegetable*. [L.]

5. Show that the analysis of an intensive equals the synthesis of an extensive whole. [C.]

6. Invent a syllogism in Barbara, and state it both in the extensive and in the intensive forms. [L.]

7. What is the place of the Major and Minor Terms in the conclusion of (*a*) an extensive, and (*b*) an intensive (comprehensive) syllogism?

8. Can the distinction of extension and intension be made to apply to the inductive syllogism? [c.]

9. Select from pp. 91 to 98 examples of the moods Celarent, Cesare, and Camenes, and state them in the intensive form.

10. What is the difference of meaning of *genus* and *species* in extent and intent? Is the extent of a notion always less as the intent is greater, and *vice versâ?*

11. Interpret the following propositions in extension and intension:—

A libel is a malicious and injurious statement.
He who believes himself to be always right in his opinion claims infallibility.
It is impossible to be and not to be.
He that can swim needs not despair to fly.

CHAPTER XVI

HYPOTHETICAL, DILEMMATIC, AND OTHER KINDS OF ARGUMENTS

1. SOME attempt will be made in the subsequent chapters on the Elements of Equational Logic to illustrate the actual and possible variety of assertions and arguments. But it will be convenient to give here a few examples of hypothetical and other arguments in the less common forms. Several subtle questions arising out of the hypothetical form of assertion are also considered with some care; but it has not been thought necessary to treat all the various forms of disjunctive and dilemmatic arguments which will be found described in almost identical terms in numerous text-books.

2. If virtue is voluntary, vice is voluntary; but virtue is voluntary; therefore so is vice. [W.]

A valid Constructive Hypothetical syllogism, equivalent to the following categorical one in Barbara:—

Beings who can be virtuous at will can also be vicious at will;
Men can be virtuous at will;
Therefore, they can be vicious at will.

3. Logic is indeed worthy of being cultivated, if Aristotle is to be regarded as infallible; but he is not: Logic, therefore, is not worthy of being cultivated. [W.]

Clearly a false hypothetical syllogism. The antecedent is, 'if Aristotle is to be regarded as infallible'; this is denied in the minor premise. In the categorical form the pseudo-argument might be stated somewhat as follows:—

Those who regard Aristotle as infallible must consider logic worthy of being cultivated;
We do not regard Aristotle as infallible;
Therefore, we do *not* consider logic, etc.

There is Illicit Process of the Major Term.

4. We are bound to set apart one day in seven for religious duties, if the fourth commandment is obligatory on us: but we are bound to set apart one day in seven for religious duties; and hence it appears that the fourth commandment is obligatory on us. [W.]

The antecedent is 'if the fourth commandment is obligatory'; the consequent is 'we are bound, etc.'; it is the consequent which is affirmed, so that the argument involves the Fallacy of Affirming the Consequent. It may be put categorically as follows: Those on whom the fourth commandment is obligatory are bound, etc.; we are bound, etc.; therefore, we are among those on whom the fourth commandment is obligatory. The fallacy is evidently that of Undistributed Middle, the pseudo-mood being **AAA** in the second figure.

5. (1) If the prophecies of the Old Testament had been written without knowledge of the events of the time of Christ, (2) they could not correspond with them exactly; (3) and if they had been forged by Christians, (4) they would not be preserved and acknowledged by the Jews: (5) they are preserved and acknowledged by the Jews, (6) and they correspond exactly with the events of the time of Christ: therefore they were (7) neither written without knowledge of those events, (8) nor were forged by Christians. [W.]

The above argument will be found to consist of two valid destructive hypothetical syllogisms woven together in statement. Thus (1) and (2) are the antecedent and consequent of the first syllogism; (6) is its negative minor, and (7) is its negative conclusion. The second syllogism has (3) and (4) for its antecedent and consequent, (5) for its negative minor premise, and (8) for its conclusion.

6. In how many ways can you state the substance of the categorical proposition 'A wolf let into the sheep-fold will devour the sheep'?

Isaac Watts, in his Essay on the Improvement of the Mind, has well pointed out the variety of expression which may be given to the same real assertion. Thus, as equivalents for the above proposition, he gives the following: 'If you let a wolf into the fold, the sheep will be devoured: The wolf will devour the sheep, if the sheep-fold be left open: If the fold be not left shut carefully, the wolf will devour the sheep: The sheep will be devoured by the wolf, if it find the way into the fold open: There is no defence of the sheep from the wolf, unless it be kept out of the fold: A slaughter will be made among the sheep, if the wolf can get into the fold.' There are various modes of hypothetically stating the result contained in the categorical original.

7. In a strictly logical point of view, ought it to be offensive to Captain Jones to say of him 'If Captain Jones does run away in battle, he will live to fight another day'?

This question touches deeply, not only the soldierly reputation of Captain Jones, but, what is much more important, the precise import of propositions. It puts forward Captain Jones as running away in battle, but it puts this forward only as a hypothesis, the result of which would be his living to fight another day. It is quite a different matter, what meaning such a proposition might be taken to imply in common life; things are often said in the form of *innuendo*. The mere coupling of a man's name with a disreputable action, even though the action were expressly denied of him, raises the question,—Why was the assertion made at all unless to bring the terms together in the mind of the hearer? If in company a gentleman were suddenly to remark 'There is not the least reason to believe that Captain Jones did run away in his last action'; here is a point blank denial of any ground for believing an assertion to that effect; yet every one would construe such a *mal-a-propos* denial as evidence of a wish to raise the question, and possibly start a rumour, which would presently take a disagreeable affirmative form. Thus we see that the logic of conversation is widely different in apparent nature from the strict logic of science; not that it is really different in the end, when thoroughly analysed. But we constantly deal with illogical, inaccurate, or even untrustworthy persons, so that we can seldom be sure that an assertion will be construed and repeated in the form which we originally gave to it. There is too much truth in the saying of Talleyrand, that words were given to us to disguise our thoughts.

8. If Brown says to Jones, 'Because Robinson is foolish you have no need to be foolish,' does Brown assert categorically that Robinson is foolish?

There can be no doubt that, in the logic of common life, Brown would be understood to make an imputation upon the wisdom of Robinson, especially if the remark was not explained by the previous course of the conversation. But in strict logic it seems very doubtful whether the conjunction 'because' should be interpreted differently from 'if,' as in the last question. 'The fact of Robinson being foolish is no reason, etc.' 'Foolishness on the part of Robinson is no reason for you being foolish.' A logical copula must not be understood to assert the physical existence and occurrence of its subject or predicate; it only asserts a relation between them.

9. If P is Q, and Q is R, it follows that P is R: but suppose it to be discovered that no such thing as Q exists, how is the truth of the conclusion, P is R, affected by this discovery?

I do not see how there is in deductive logic any question about existence. The inference is to the effect that if the propositions *P* is *Q* and *Q* is *R* are true, then the conclusion *P* is *R* is true. The non-existence of *Q* may possibly render one or both premises materially false, in which case the reasoning vanishes, but is not logically defective. If I argue, for instance, that satyrs are creatures half man and half goat; and creatures half man and half goat are very hideous, therefore satyrs are very hideous; the reasoning is equally good whether satyrs exist or not. We cannot, of course, say that the conclusion is materially true, if there be no objects to which the material truth can apply. But if I argue that satyrs are creatures half man and half goat, and such creatures exist in Thessaly, therefore, satyrs exist in Thessaly; in this case the non-existence of the middle term would affect the material truth of the second premise, and, if this be held false, we cannot affirm the material truth of the conclusion.

I ought to add that De Morgan in more than one place assumes that the middle term must have existence, or even *objective* existence; thus he says (*Syllabus*, p. 67): 'In all syllogisms the *existence* of the middle term is a *datum*,' etc. This is one of the few points in which it is possible to suspect him of unsoundness.

The student may refer to Hamilton's *Lectures*, vol. iii. pp. 454-5, and p. 459, on 'Sophisms of Unreal Middle'; see also Whately's *Analytical Outline*, § 3.

10. Lias lies above red sandstone; red sandstone lies above coal; therefore lias lies above coal. [W.]

This is one of many examples to be found in the logic books of arguments which simulate the syllogistic form. It is often said that they can be solved syllogistically; but certainly this cannot be done by the ordinary rules and processes of the syllogism. The most that we can get, even by substitution, is that 'Lias lies above what lies above coal.' The fact is that the argument is really a mathematical one, involving simple equations. It is precisely similar to one which has been thus treated by Professor F. W. Newman (*Miscellanies*, 1869, p. 28), and Mr. J. J. Murphy. The former of these logicians, as quoted by the latter, remarks—'The argument *Lead is heavier than silver; Gold is heavier than Lead: therefore Gold is heavier than Silver*,[1] brings to the mind conviction as direct as the simplest of syllogisms. To say that its validity *depends on its being reducible* to syllogism, is wholly unplausible: for to effect the reduction, you have to make changes of form at least as hard to accept as the direct argument: and when you have got your syllogisms, they are more complicated and cumbrous than the argument as it stands.'

Mr. Murphy (*The Relation of Logic to Language*: Belfast Natural History and Philosophical Society, 17th February 1875) treats the argument simply as a question of quantity, thus—

'Call the weights of gold, lead, and silver respectively x, y, and z: then

$$x = y + p$$
$$y = z + q$$
$$x = z + q + p.$$

In the old logic, the foregoing conclusion could be drawn only by means of the following syllogism:—

That which is greater than the greater is greater than the less:

The weight of gold is greater than that of lead, and the weight of lead greater than that of silver:

[1] Newman has inadvertently written *Lead is heavier than Gold*, which is wrong as to fact.

Therefore the weight of gold is greater than that of silver.

Considered as fact all this of course is true, but considered as logic it is wrongly stated. That which is here stated as the major premise is really the syllogistic canon. It is not merely a general truth, like the truth that all matter gravitates, but a logical principle, lying as near to the first principles of the science as the axiom that a part of a part is a part of the whole.

We have only to assume x to be the height of lias, y the height of red sandstone, and z of coal above any one fixed datum line, and the same equations represent the argument at the head of this section.

It may be added that Reid was doubtless right in denying that we argue syllogistically when we infer that because A and C are both equal to B they are equal to each other. We may throw it into the form, 'Things equal to the same are equal to each other; A and C are things equal to the same, therefore they are equal to each other.' But this is a delusive syllogism. The inference is really accomplished in obtaining the major premise. The inferences of equality are prior to and simpler than the inferences of logic, and the attempt of Herlinus and Dasypodius to throw Euclid into the syllogistic form has been rightly ridiculed, because it is an attempt to prove the more simple and self-evident by means of the more complex.

Some remarks on this point will be found in De Morgan's *Second Memoir on the Syllogism*, 1850, pp. 50, 51; his *Fourth Memoir*, 1860, p. 8, etc.; in Mr. Murphy's paper quoted above; and in Hallam's remarkable note to Section 129 of Vol. III., Chapter III., of his *Introduction to the Literature of Europe* (1st Ed. p. 288; 5th Ed. p. 111).

CHAPTER XVII

EXERCISES IN HYPOTHETICAL ARGUMENTS

1. (1) IF he is well, he will come: he is not well: therefore he will not come.

   (2) If he is well, he will come: he will come: therefore he is well. [H.]

   (3) I am sure he will not come, for he is not well; and if well he would come.

   (4) He will write if he is well; but as he is not well, therefore he will not write.

Analyse the above arguments and point out which are fallacious, and why.

2. Into how many forms of expression can you throw the matter of this proposition? 'Sulphuric acid combined with calcium produces gypsum.'

3. Throw into the form of hypothetical propositions the following disjunctives—

   (1) Either the Claimant is Orton, or many witnesses are mistaken.

   (2) The tooth of a mammalian is either an incisor, canine, bicuspid, or molar tooth.

4. Under which of the commonly recognised forms of syllogism would you bring the following?—

If *A* is *B*, *C* is *D*;
If *C* is *D*, *E* is *F*;
Therefore, If *A* is *B*, *E* is *F*. [C.]

5. Are hypothetical propositions capable of conversion? If so, convert these—

(1) If it has thundered it has lightened.
(2) Unless it has lightened it has not thundered.

6. Which of the following arguments are logically correct?—

(1) *A* is *B*, if it is *C*; it is not *C*, therefore it is not *B*.
(2) *A* is not *B* unless it is *C*; as it is not *C*, it is not *B*.
(3) If *A* is not *B*, *C* is not *D*; but as *A* is *B*, it follows that *C* is *D*.
(4) *A* is not *B*, if *C* is *D*; *C* then is not *D*, for *A* is *B*.

7. If the Hypothetical *Modus Ponens* and *Modus Tollens* are taken as corresponding to the Categorical First and Second Figures, and their typical forms to the Moods Barbara and Camestres, respectively, what other forms of the respective Hypothetical *Modi* would correspond to the other moods of the respective Categorical Figures? [R.]

8. If *A* is true, *B* is true; if *B* is true, *C* is true; if *C* is true, *D* is true. What is the effect upon the other assertions of supposing successively that (1) *D* is false; (2) that *C* is false; (3) that *B* is false; (4) that *A* is false?

9. Analyse the following arguments and estimate their validity.

(1) I shall see you if you do not go; but as you are going I shall not.

(2) The Penge convicts were guilty of murder, if, after long continued neglect at their hands, Harriet Staunton died.

(3) Since the virtuous alone are happy, he must be virtuous if he is happy, and he must be happy if he is virtuous.

(4) If there were no dew the weather would be foul: but there is dew; therefore the weather will be fine. [O.]

(5) If there are sharpers in the company we ought not to gamble; but there are no sharpers in the company; therefore we ought to gamble. [E.]

(6) 'I could then only be accused with justice of acting contrary to my law, if I maintained that Muraena purchased the votes, and was justified in doing so. But I maintain that he did not buy the votes, therefore, I do nothing contrary to the law.'—Cicero, Pro L. Muraena, c. iii. (See Devey's *Logic*, 1854, p. 133.)

10. State in the form of a disjunctive argument the matter of the First Book of Samuel, chapter xvi. verses 6-13.

11. Examine the question whether hypothetical and disjunctive arguments are reducible to the forms of the categorical syllogism.

12. Dilemmatic arguments are more often fallacious than not. Why is this? [C.]

13. Investigate the logical position of the parties to the following colloquy from Clarissa Harlowe: '*Morden*—But *if* you have the value for my cousin that you say you

have, you must needs think——. *Lovelace*—You must allow me, sir, to interrupt you. *If* I have the value I *say* I have! I hope, sir, when I *say* I have that value, there is no cause for that *if*, as you pronounced it with an emphasis. *Morden*—Had you heard me out, Mr. Lovelace, you would have found that my *if* was rather an *if* of inference than of doubt.'

This passage is quoted and discussed by Professor Croom Robertson in *Mind*, 1877, vol. ii. pp. 264-6.

CHAPTER XVIII

THE QUANTIFICATION OF THE PREDICATE

1. As explained in the preface, I have thought it well to discuss and illustrate in this book of exercises, the forms of logical expression and inference recognised by Dr. Thomson and Sir W. Hamilton. These correspond in most cases with what De Morgan represented under different systems of notation. They also correspond to some of the expressions and arguments current in ordinary life. Although in a scientific point of view it is far better to eliminate the logical will-of-the-wisp 'some,' yet the student is obliged to make himself acquainted with the pitfalls into which it is likely to lead him.

It is assumed that the reader has studied the brief account of the Quantification of the Predicate given in the 22nd of the *Elementary Lessons in Logic*, and he is recommended to read, on the same subject, either Thomson's Outline, or else Bowen's account of Hamilton's Logic (Bowen's *Logic*, Chapter VIII.) The study of De Morgan's and Hamilton's own writings is a more arduous and hazardous undertaking.

The following are the eight kinds of propositions recognised by Hamilton, as described by Dr. Thomson.

Sign.	*Affirmative.*	*Negative.*	*Sign.*
U	All X is all Y.	No X is Y.	**E**
I	Some X is some Y.	Some X is not some Y.	ω
A	All X is some Y.	No X is some Y.	η
Y	Some X is all Y.	Some X is no Y	**O**

2. Indicate by the technical symbols the quantity and the quality of the following propositions :—

   (1) All primary forces are attractive.
   (2) All vital actions come under the law of habit, and none but vital actions do.
   (3) The best part of every man's education is that which he gives himself.
   (4) Only ungulate animals have horns.
   (5) Mere readers are very often the most idle of human beings.
   (6) Most water-breathing vegetables are flowerless. [P.]

(1) Is clearly a universal affirmative (**A**).

(2) As regards its first part is also **A**; but the exclusive addition, 'None but vital actions do,' means that 'all not vital actions do not.' The two parts together yield a proposition in **U**, 'all vital actions are all that come under the law of habit.'

(3) The 'best part,' being a superlative, is a singular term, and so is the predicate 'that part which, etc.' Hence the proposition is an identity in **U**.

(4) An exclusive proposition equivalent to 'all not ungulate animals have no horns,' which is the contrapositive of, and equivalent to, 'all horned animals are ungulate.'

(5) Means that a great many mere readers are, etc., and is in the form **I**.

(6) Is also a particular affirmative proposition.

3. Does not the proposition **Y** of Thomson imply **O**, that is to say, does not 'some *P* is all *Q*' imply that 'some *P* is not *Q*'?

This seems very plausible, because if some *P* makes up the whole of *Q*, there is, so to say, no room left in *Q*'s sphere for any more *P*s, the remainder of which must therefore be not *Q*. This argument, however, overlooks the fact that the 'some *P*' in question may possibly be the whole of *P*, so that there may be no remainder excluded from *Q*.

4. Is the proposition 'Some men are animals' true? [E.]

The proposition is true or untrue *materially* according to the sense we put upon this troublesome word *some*. If we take it to mean 'one or more it may be all,' the proposition is true in fact, but of course states less than is known to every one.

We must carefully distinguish between the strict and necessary logical interpretation of 'some,' and that which applies in colloquy. De Morgan says (*Formal Logic*, p. 4), 'In common conversation the affirmation of a part is meant to imply the denial of the remainder. Thus, by "some of the apples are ripe," it is always intended to signify that "some are not ripe." There is no difficulty in providing in formal logic for this use of the word by stating explicitly the two propositions which are colloquially merged into one. Thus 'some of the apples are ripe' is really **I** + **O**.

5. What results would follow if we were to interpret 'some *A*s are *B*s' as implying that 'some other *A*s are not *B*s'?

The proposition 'some *A*s are *B*s' is in the form **I**, and according to the table of opposition (p. 31) **I** is true if **A** is true; but **A** is the contradictory of **O**, which would be the form of 'some other *A*s are not *B*s.' Under such circumstances **A** could never be true at all, because its truth would involve the truth of its own contradictory, which is absurd.

Briefly—If **A** is true, **I** is true; and if **I** implies **O**, then **A** implies the truth of its own contradictory **O**.

Several logicians have come to grief over this troublesome word, notably Sir W. Hamilton, who in holding that 'some' is formally exclusive of all and none, throws all logical systems into confusion. Woolley commits the same great mistake in saying (p. 77), 'In every particular proposition, therefore, the *affirmative and negative mutually imply each other*: if *only some A* is *B*, then some *A* is *not B*, and *vice versâ*.'

6. Explain the precise meaning of the proposition 'Some *X*s are not some *Y*s' (the proposition ω of Thomson). What is its contradictory? Give your opinion of its importance.

This is one of the eight forms of proposition which Hamilton, in pursuance of the thoroughgoing quantification of the predicate, introduced into his system. Now, if 'some *Y*' means 'any some *Y*,' that is to say, if the 'some' is undetermined and may be any where in the sphere of *Y*, this proposition does not differ from 'some *X* is not any *Y*,' which is the proposition **O** of the old Aristotelian Logic. But if 'some *Y*' is a determinate part of the class *Y*, less than the whole, then the proposition becomes a mere empty truism; for, however *X* and *Y* may be related, some part of *X* will be different from some part of *Y*. Thus all equilateral triangles are all equiangular triangles, yet some equilateral triangles are not some equiangular ones. If all John Jones' sons are Rugby boys, yet some of John Jones' sons are not some Rugby boys. We see that this proposition ω is consistent with all the other propositions of the system, in all cases, as De Morgan remarks (*Syllabus*, p. 24), 'in which either *X* or *Y* has two or more instances in existence: its contrary is "*X* and *Y* are singular and identical; there is but one *X*, there is but one *Y*, and *X* is *Y*."' A system which offers an assertion and denial which cannot be contradicted in the same system carries its own condemnation with it, as well observed by De Morgan.

Archbishop Thomson also rejected this form of proposition. He says: 'If I define the composition of common salt by saying, "common salt is chloride of sodium," I cannot prevent another saying that "*some* common salt is not some chloride of sodium," because he may mean that the common salt in *this* salt-cellar is not the chloride of sodium in *that*. A judgment of this kind is spurious upon two grounds: it denies nothing, because it does not prevent any of the modes of affirmation; it decides nothing, inasmuch as its truth is presupposed with reference to any pair of conceptions whatever.' (*Outline of the Laws of Thought*, 1860, § 79, p. 137.)

SPALDING, pp. 83, 97-102, etc., symbolises the proposition ω, by $\frac{1}{2}$ **O**.

In an examination, candidates almost invariably say that all *X*s are *Y*s, or all *X*s are all *Y*s, is the contradictory of some *X*s are not some *Y*s; and De Morgan (1863, p. 4) speaks of an unnamed logical author who spoiled his work with a like blunder.

The chief interest of this proposition ω arises from its important bearing upon the value of Hamilton's *System of Logic*, and his position as a logician. Hamilton insisted upon the *thoroughgoing* quantification of the predicate, which means the recognition and employment of *all* the eight propositions which the introduction of the quantified predicate renders conceivable. Thus was the key-stone to be put into the arch of the Aristotelic logic. But if, as Thomson and De Morgan seem to me to have conclusively shown, this proposition, ω, is valueless and absurd, the key-stone crumbles and the arch collapses. The same ruin does not overtake De Morgan's system, because his eight propositions are not all the same as those of Hamilton; nor does it affect in any appreciable degree the views of Thomson and George Bentham, who did not insist upon the thoroughgoing quantification of the predicate.

De Morgan has admirably expressed the inherent ambiguity of this word. He says (Fifth Memoir on the Syllogism, 1863, p. 4), '"He has got some apples" is very clear: ask the meaning of "he has not got some apples," in a company of educated men, and the apples will be those of discord. Some will think that he may have one apple; some that he has no apple at all; some that he has not got some particular apples or species of apples.'

The subject of particular propositions may be pursued in Spalding's Logic, 1857, p. 172, and elsewhere; Shedden's Logic; Hughling's Logic of Names, 1869, p. 31; Thomson's Outline, fifth edition, section 77; Hamilton's Lectures, vol. iv. pp. 254, 279; Devey, 1854, pp. 90-94; De Morgan, 1863, Fifth Memoir.

Mr. A. J. Ellis is particularly exact in his treatment of this question in his articles in the *Educational Times*, 1878.

7. Solly says (p. 73) 'If the premises are "some *B* is *A*, some *C* is not *B*," the reason may logically deduce that some *C* is not some *A*. But this conclusion is not in one of the four legitimate forms.' Is the argument valid in the quantified syllogism, and if so, in what mood?

The propositions are as follow:—

Some *B* is some *A*	**I**
Some *C* is not (any *B*)	**O**
Some *C* is not some *A*	ω

The middle term is distributed once in the minor premise, and, as both terms of ω are particular, there can be no illicit process. One premise is negative, and so is the conclusion. No rule of the syllogism is broken, and the argument is therefore valid. It appears as **I O** ω in the sixth mood of the first figure of Thomson's table.

8. Which of the following conjunctions of propositions make valid syllogisms? In the case of those which you regard as invalid, give your reasons for so treating them.

First Figure.	Second Figure.	Third Figure.
E Y O	**U O** ω	**A** ω **O**
A E E	η **U O**	**Y E O**
I U η		

[C.]

The pseudo-mood **A E E** in the first figure gives illicit process of the major term, because the conclusion **E** distributes its predicate, and the major premise **A** does not. The pseudo-mood **I U** η draws a negative conclusion, η, from two affirmative premises, but is by oversight given in Thomson's Table of Modes, figure 1, mode xii., second negative form. It is an obvious misprint for **I E** η. (*Outline of the Laws of Thought*, section 103, 5th ed., p. 188.) In the table as reprinted in the *Elementary Lessons in Logic*, p. 188 (accidentally the same page as in Thomson !), the error was corrected in the fifth and later editions. It was pointed out to me by Mr. A. J. Ellis.

E Y O is valid in the first figure.

In the second figure **U O** ω breaks no rule, but the conclusion instead of being ω (some *X*s are not some *Z*s), might have been in the stronger form **O** (some *X*s are not any *Z*s). The moods **U O O** and **U** ω ω appear in Thomson's table, column 4, though **U O** ω does not. The mood η **U O** is valid.

In the third figure **A** ω **O** is subject to illicit process of the major term, since the conclusion **O** distributes its predicate, which is the undistributed predicate of **A** in the major premise. **Y E O** is not subject to the same objection, because **Y** distributes its predicate; but, in this last case, the conclusion is weakened, and might have been **E**; hence **Y E E** appears in Thomson's table, tenth mood of third figure, and **Y E O** does not appear.

9. In what mood is the following argument: Aliquod trilaterum est aequiangulum; omne triangulum est (omne) trilaterum; ergo, aliquod triangulum est aequiangulum?

The first premise, 'some trilateral figure is an equiangular figure,' is plainly a proposition in **I**; the second, 'all triangles are *all* trilateral figures,' is as plainly a doubly universal proposition in **U**; the conclusion, 'some triangular figure is equiangular,' is in **I**. The middle term, trilaterum, is distributed in the minor premise, though not in the major; there is no illicit process, nor other breach of the syllogistic rules, so that the argument is a valid syllogism in the mood **I U I** of the first figure. It appears as the twelfth mood in the first column of Thomson's table of moods. See, however, Baynes' *New Analytic*, 1850, pp. 126-7, whence this example is taken.

10. Does the following argument fall into any valid mood of the syllogism?

Some man is all lawyer;

Any lawyer is not any stone;

therefore, Some man (*i.e.* lawyer) is not any stone (*i.e.* all the rest are stone).

This example is taken by De Morgan (1863, p. 10) as a case of Hamilton's mood IV. *b*, as stated in his *Lectures on Logic*, vol. iv. p. 287, thus, 'A term parti-totally co-inclusive, and a term totally co-exclusive, of a third, are parti-totally co-exclusive of each other.' It was called by De Morgan the 'Gorgon Syllogism,' alluding, I presume, to the petrifying effect it produces upon all mankind who are not lawyers. It is plainly in the mood **E Y** ω, and though it does not appear in Thomson's table, may be considered a weakened form of **E Y O**, the seventh negative mood of the first figure. The point of the matter, however, is that Hamilton, in his later writings, proposed to depart from the Aristotelian sense of the mark of particular quantity *some*. As stated in his Lectures, vol. iv. p. 281, the view which he wished to introduce is that *some* should mean 'some at most,—some only,—some not all.' But, if we apply this meaning of 'some' to the conclusion of the

Gorgon Syllogism, it produces the ridiculous result that, *though lawyers are not stone, all the rest of mankind are stone.* De Morgan is unquestionably correct, and this Gorgon Syllogism brings to ruin Hamilton's 'long adequately tested and matured' system.

The particulars of the discussion between De Morgan and Professor Spencer Baynes about this Gorgon Syllogism, and kindred matters, may be found in the *Athenæum* of 1861 and 1862 and elsewhere.

It is curious that De Morgan states the Gorgon Syllogism differently in the *Athenæum* of 2nd November 1861, p. 582, and in his Fifth Memoir on the Syllogism, p. 10; but the difference is not material to the final issue.

11. 'The month of May has no "R" in its name; nor has June, July, or August: all the hottest months are May, June, July, and August: therefore, all the hottest months are without an "R" in their names.'

This is Whately's example No. 117, and as he refers the student to Book IV., Chap. I., § 1, which treats of induction, he evidently regards it as an Inductive Syllogism. It would have been referred by Hamilton to the Thomsonian mood **E U E**, the minor premise being treated as a doubly universal proposition. There can be no doubt, however, that the minor is really disjunctive, thus: A hottest month is either May, or June, or July, or August. The major is a compound sentence, comprising four separate propositions, 'May has no R in its name,' 'June has no R,' etc. (See *Elementary Lessons*, Lesson XXV., p. 215.)

CHAPTER XIX

EXERCISES ON THE QUANTIFICATION OF THE PREDICATE

1. EXPRESS carefully, in full logical form, with quantified subjects and predicates, the following propositions; assign the Thomsonian symbol in each case:—

(1) Thoughts tending to ambition, they do plot unlikely wonders.

(2) Fools are more hard to conquer than persuade.

(3) Heaven has to all allotted, soon or late,
Some happy revolution of their fate.

(4) Justice is expediency.

(5) This is certainly the man I saw yesterday.

(6) Man is the only animal with ears that cannot move them.

(7) Wisdom is the habitual employment of a patient and comprehensive understanding in combining various and remote means to promote the happiness of mankind.

(8) It is among plants that we must place all the Diatomaceae.

(9) When the age is in the wit is out.

(10) Every man at forty is either a fool or a physician.

(11) Some men at forty are neither fools nor physicians.

(12) Some men at forty are both fools and physicians.

(13) L'État c'est moi, as Louis the Fourteenth used to say.

(14) There are no coins excepting those made of metal, if we overlook a few composed of porcelain, glass, or leather.

(15) Antisthenes said δεῖν κτᾶσθαι νοῦν ἢ βρόχον.

(16) All animals which have a language have a voice, but not all which have a voice have a language.

(17) The elephant alone among mammals has a proboscis.

(18) Prudence is that virtue by which we discern what is proper to be done under the various circumstances of time and place.

(19) Whatever is, is right.

(20) There are arguments and arguments.

(21) A dispute is an oral controversy, and a controversy is a written dispute.

(22) There beth workys of actyf lyf othere gostly othere bodily.

(23) The only Roman who gave us a summary of Aristotle was the only Roman who gave us a summary of Euclid.

(24) Zenobia declared that the last moment of her reign and of her life should be the same.

(25) As it asketh some knowledge to demand a question not impertinent, so it requireth some sense to make a wish not absurd.

(26) Mankind consists of dark men and fair men.

(27) To say that Mr. Raffles was excited was only another way of saying that it was evening.

(28) Though all well educated men are not discoverers, all discoverers are well educated men.

(29) No man is esteemed for gay garments but by fools and women.

(30) Quand celui qui ecoute n'entend rien, et quand celui qui parle n'entend plus, c'est la metaphysique.

(31) Friendship finds men equal or makes them so.

(32) I can fly or I can run.

(33) A man is an ill husband of his honour that entereth into an action, the failing wherein may disgrace him more than the carrying of it through can honour him.

(34) Scribendi recte sapere est principium et fons.

(35) Tools are only simple machines, and machines are only complicated tools.

(36) The wise man knows the fool, but the fool knows not the wise man.

(37) It is scandalous that he who sweetens his drink by the gift of the bees, should by vice embitter Reason, the gift of the Gods.

(38) *A* and *B* and *C* and *D*, etc., etc., wear black coats on Sundays; in fact every man I know does so.

(39) All the Apostles were Jews, because this is true of Peter, James, John, and every other Apostle.

(40) A dose of arsenic is given to a living healthy dog. Soon after the dog dies. Arsenic is therefore a poison.

2. How can a chain of reasoning, founded on circumstantial evidence, be represented in syllogistic form? [E.]

3. Having special regard to the logical sense of 'some,' what do you think of the validity of the following argument (Thomson's Syllogistic Mood **A E** η)?

All *Y* is some *X*;
No *Z* is any *Y*;
Therefore, No *Z* is some *X*.

4. 'We have been assured that "all X is some Y" is contradicted by "all Y is some X," a proposition which cannot be made good except by *some* being declared *not all*.' (De Morgan, Third Memoir, 1858, p. 24.) Investigate this point.

5. Take 'stone' and 'solid' as subject and predicate, and convince yourself that the proposition in ω, 'some stone is not some solid,' cannot be contradicted by any propositions of the forms **U**, **A**, **I**, **Y**, **E**, **O**, η, having the same subject and predicate.

6. Write out the various judgments, including **U** and **Y**, which are logically opposed to the judgment, 'No puns are admissible.' State in the case of each judgment thus formed what is the kind of opposition in which it stands to the original judgment, and also the kind of opposition between each pair of the new judgments. [C.]

7. 'The judgment, "No birds are *some* animals," is never actually made because it has the semblance only, and not the power of a denial.' Examine this statement. [P.]

8. Draw inferences from the following:

If Sir Thomas was imbecile, then Oliver was right; and unless Sir Thomas was imbecile, Oliver was not wrong. [P.]

9. Examine the following arguments; in those which are false point out the nature and name of the fallacy; arrange those which are valid syllogisms in the usual form, and give the symbolic description of the mood.

(1) All the householders in the kingdom, except women, are legally electors, and all the male householders are precisely those men who pay poor-rates; it follows that all men who pay poor-rates are electors.

(2) All the times when the moon comes between the earth and the sun, are the sole cases of a solar eclipse; the 11th of February is not such a time; therefore, the 11th of February will exhibit no eclipse of the sun. [THOMSON.]

(3) All men are mortals, and all mortals are all those who are sure to die; therefore, all men are all those who are sure to die.

(4) The Claimant is unquestionably Arthur Orton: for he is Castro, who is the same person as Arthur Orton.

10. Which of the following moods are legitimate, and in what figures:—**E Y O, Y A A, Y A Y, I Y I, Y Y Y, A E E?** [M.]

11. Examine the validity of the following moods:

Figure I.	Figure II.	Figure III.	
U A U	**A A A**	**Y E E**	
Y O O	**A Y Y**	**O Y O**	[C.]

12. Exemplify any of the following moods, and determine in how many figures each is valid: **U U U, I U I, Y U Y,** η **U** η, ω **U** ω.

CHAPTER XX

EXAMPLES OF ARGUMENTS AND FALLACIES

THIS chapter contains a large collection of examples of Arguments and Fallacies collected from many sources. They form additional illustrations and exercises to supplement what are given in the previous chapters. The student is to determine in the case of each example whether it contains a valid or fallacious argument. In the former case he is to throw the example into a regular form, and assign the technical description of that form, whether a mood of the categorical syllogism, or of the hypothetical or disjunctive syllogism, etc. In some examples two or more syllogisms, or two or more different forms of reasoning, will be complicated together. They must of course be analysed and exhibited separately.

When the existence of fallacy is suspected, the student must endeavour to reduce this to a distinct paralogism or breach of the syllogistic rules, exhibiting the pseudo-mood or pseudo-form of reasoning. In many cases, however, the fallacy may be of the kinds described in the Aristotelian text-books as *Semi-logical* or *Material.* These fallacies have been explained in the Elementary Lessons (Lessons XX. and XXI.), but for convenience of reference a simple list of the kinds of fallacies is given below. It has not been found practicable to undertake in this book a full exemplification of the subject of Fallacies. The student is therefore referred to the Elementary Lessons named, or to any of the following writings on the subject :—

De Morgan's *Formal Logic*, Chapter XIII., as amusing as it is accurate and instructive ; Whately's *Logic*, Book III., perhaps the best and most interesting part of this celebrated text-book ; Edward Poste's edition of *Aristotle on Fallacies*.

Paralogisms

1. Four terms. Breach of Rule I.
2. Undistributed Middle. Breach of Rule III.
3. Illicit Process of Major or Minor Term. Breach of Rule IV.
4. Negative premises. Breach of Rule V.
5. Negative Conclusion from affirmative premises, and *vice versâ*. Breach of Rule VI.

Breaches of Rules VII. and VIII. can be resolved into one or other of the above.

Semi-logical Fallacies	*Material Fallacies*
1. Equivocation.	1. Accident.
2. Amphibology.	2. Converse Fallacy of Accident.
3. Composition.	3. Irrelevant Conclusion.
4. Division.	4. Petitio Principii.
5. Accent.	5. Non Sequitur.
6. Figure of Speech.	6. False Cause.
	7. Many Questions.

1. France, having a warm climate, is a wine-producing country. [E.]

2. Livy describes prodigies in his history ; therefore he is never to be believed. [E.]

3. All the metals conduct heat and electricity; for iron, lead, and copper do so, and they are (all) metals. [E.]

4. A charitable man has no merit in relieving distress, because he merely does what is pleasing to himself. [E.]

5. What is the result of all this teaching? Every day you hear of a fraud or forgery, by some one who might have led an innocent life, if he had never learned to read or write. [E.]

6. The use of ardent spirits should be prohibited by law, seeing that it causes misery and crime, which it is one of the chief ends of law to prevent. [E.]

7. Pious men only are fit to be ministers of religion; some ignorant men are pious; therefore ministers of religion may be ignorant men. [L.]

8. No punishment should be allowed for the sake of the good that may come of it; for all punishment is an evil, and we are not justified in doing evil that good may come of it. [E.]

9. We know that God exists because the Bible tells us so; and we know that whatever the Bible affirms must be true because it is of Divine origin. [E.]

10. The end of punishment is either the protection of society or the reformation of the individual. Capital punishment ought therefore to be abolished. It does not in fact prevent crimes of violence, and so fails to protect society, while on the other alternative it is absurd. [E.]

11. The glass is falling; therefore we may look for rain. [E.]

12. This is a dangerous doctrine, for we find it upheld by men who avow their disbelief in Revelation.

13. If there is a demand for education, compulsion is unnecessary. [E.]

14. Actions that benefit mankind are virtuous; therefore it is a virtuous action to till the ground.

15. Slavery is a natural institution; therefore it is wrong to abolish it.

16. No fool is fit for high place; John is no fool; therefore John is fit for high place. [E.]

17. He is not a Mahometan, for no Mahometan holds these opinions. [E.]

18. Mind is active; matter is not mind; therefore matter is not active. [E.]

19. He must be a Mahometan, for all Mahometans hold these opinions. [E.]

20. If we are to believe philosophers, knowledge is impossible, for one set of them tell us that we can know nothing of matter, and another that we can know nothing of mind. [O.]

21. Old age is wiser than youth; therefore we must be guided by the decisions of our ancestors. [O.]

22. Political assassins ought not to be punished, for they act according to their consciences. [O.]

23. If education is popular, compulsion is unnecessary; if unpopular, compulsion will not be tolerated. [O.]

24. Nations are justified in revolting when badly governed, for every people has a right to good government. [E.]

25. These two figures are equal to the same figure, and therefore to each other.

26. Opium produces sleep, for it possesses a soporific virtue. [E.]

27. Wealth is in proportion to value, value to efforts, efforts to obstacles: ergo, wealth is in proportion to obstacles.

28. When Croesus has the Halys crossed, a mighty army will be lost.

29. A manor cannot begin at this day, because a court-baron cannot now be founded.

30. Poeta nascitur, non fit; how absurd it is then to teach the making of Latin verses!

31. Aio te Aeacida, Romanos vincere posse.

32. Every rule has exceptions; this is a rule, and therefore has exceptions; therefore there are some rules that have no exceptions. [E.]

33. All that perceives is mind; the existence of objects consists in being perceived; therefore the existence of objects necessarily depends on mind. [E.]

34. Some objects of great beauty merely please the eye; for instance, many flowers of great beauty, and accordingly they answer no purpose but to gratify the sight. [H.]

35. A miracle is a violation of the laws of nature; and, as a firm and unalterable experience has established those laws, the proof against a miracle, from the very nature of the fact, is as entire as any argument from experience can possibly be. [E.]

36. The imagination and affections have a close union together. The vivacity of the former gives force to the latter. Hence the prospect of any pleasure with which we are acquainted affects us more than any other pleasure which we may own superior, but of whose nature we are wholly ignorant. [E.]

37. Common salt consists of a metal and a metalloid, for it consists of sodium and chlorine, of which one is a metal, and the other a metalloid. [E.]

38. 'The truth is, that luxury produces much good. A man gives half-a-guinea for a dish of green peas; how much gardening does this occasion?' (Dr. Johnson.) [O.]

39. Nothing can be produced; for what exists cannot be produced, as it is already in existence, and what does not exist cannot be produced, as, since it is not in existence, nothing can happen to it. [E.]

40. The earth's position must be fixed, if the fixed stars are seen at all times in the same situations: now the fixed stars are not seen at all times in the same situations; therefore the earth's position is not fixed. [E.]

41. Protective laws should be abolished, for they are injurious if they produce scarcity, and they are useless if they do not. [E.]

42. All who think this man innocent think he should not be punished; you think he should not be punished, therefore you think him innocent. [E.]

43. If we are disposed to credit all that is told us, we must believe in the existence, not only of one, but of two or three Napoleon Buonapartes; if we admit nothing but what is well authenticated, we shall be compelled to doubt the existence of any. How, then, can we be called upon to believe in the one Napoleon Buonaparte of history? [O.]

44. We cannot know what is false, for knowledge cannot be deceptive, and what is false is deceptive. [E.]

45. A necessary being cannot be the effect of any cause; for, if it were, its existence would depend upon the existence of its cause, and therefore would not be necessary. [E.]

46. The table we see seems to diminish as we move from it; but the real table suffers no change: it was not, therefore, the table itself, but only its image, that was present to the mind.

47. The existence of sensations consists in being perceived: all objects are really collections of sensations; therefore their existence consists in being perceived. [E.]

48. If the earth were of equal density throughout, it would be about $2\frac{1}{2}$ times as dense as water: but it is about $5\frac{1}{2}$ times as dense; therefore the earth must be of unequal density.

49. Whatever is conditioned must depend on some cause external to itself: this world is conditioned by time and space; therefore this world depends upon some cause external to itself. [E.]

50. It sometimes happens that an electrical current is excited, where none but magnetic forces are directly called into play; for such a current, in certain cases, is excited in an electric non-conductor by moving a magnet to or away from it. [R.]

51. The Quaker asserts that if men were true Christians, and acted upon their religious principles, there would be no need of armies. Hence he draws the conclusion that a military force is useless, and being useless, pernicious.

52. Detention implies at least possession; for detention is natural possession.

53. Nothing can be conceived without extension: what is extended must have parts; and what has parts may be destroyed. [O.]

54. Had an armistice been beneficial to France and Germany, it would have been agreed upon by those powers: but such has not been the case; it is plain therefore that an armistice would not have been advantageous to either of the belligerents. [O.]

55. By the law of nature as soon as Adam was created he was governor of mankind, for by right of nature it was due to Adam to be governor of his posterity. [O.]

56. When men are pure, laws are useless; when men are corrupt, laws are broken.

57. There are many arguments which we recognise as

valid which it is impossible to express in a syllogistic form: therefore the syllogism is valueless as a test of truth. [O.]

58. No man can be a law to himself; for law implies a superior who gives the law and an inferior who obeys it; but the same person cannot be both ruler and subject. [O.]

59. It is injustice to the intellect of women to refuse them the suffrage; for the reigns of many queens, as our own Elizabeth or Anne, have been famous for literary productions. [O.]

60. To allow every man unbounded freedom of speech must be advantageous to the State, for it is highly conducive to the interests of the community that each individual should enjoy an unlimited liberty of expressing his sentiments. [O.]

61. Your sorrow is fruitless, and will not change the course of destiny. Very true, and for that very reason I am sorry. [O.]

62. Because some individuals have in their very childhood advanced beyond the youthful giddiness and debility of reason, it only needs a proper system of education to make other young people wise beyond their years. [O.]

63. Haste makes Waste, and Waste make Want; therefore a man never loses by delay. [O.]

64. If peace at any price is desirable, war is an evil; and as war is confessedly an evil, peace at any price is desirable. [O.]

65. The two propositions, 'Aristotle is living,' and, 'Aristotle is dead,' are both intelligible propositions; they are both of them true or both of them false, because all intelligible propositions must be either true or false. [E.]

66. No form of democracy is subject to violent revolutions, because it never excludes the mass of the people from political power. [E.]

67. The student of History is compelled to admit the Law of Progress, for he finds that Society has never stood still. [E.]

68. It is fated that I shall or that I shall not recover, in either of which cases the employment of a physician is useless, and therefore inexpedient. [E.]

69. The assertion that men much occupied in public affairs cannot have time for literary occupations is disproved by such instances as Julius Caesar, Alfred, Lord Bacon, Sir G. C. Lewis, the Earl of Derby, Mr. Gladstone, and the late Emperor of the French.

70. Whatever had a beginning in time has limits in space; the universe has no beginning in time: therefore the universe has no limits in space.

71. The mollusc is an aggregate of the second order; for there is no sign of a multiplicity of like parts in its embryo.

72. The farmers will not pay in rent more than the net produce of their farms; for no trading class will continue a losing business. [L.]

73. The knowledge of things is more improving than the knowledge of words. The study of Physics must therefore be more improving than the study of Languages. [E.]

74. The moral world is far from being so well governed as the material; for the former, although it has *its* laws, which are invariable, does not observe these laws so constantly as the latter. [P.]

75. England has a gold coinage and is a very wealthy country; therefore, it may be inferred that other countries having a gold coinage must be wealthy.

76. Most parents are the best judges of the age at which their children should be sent to school, and as it is un-

desirable to interfere with those who are the best judges of their children's interest, it follows that parents should not be compelled to send their children to school.

77. Among the bodies which do not move in elliptic orbits are some of the comets; but all bodies which do move in elliptic orbits return periodically; hence, some bodies which return periodically cannot be comets.

78. Some rate-payers are clearly not fit for their duties; for all male rate-payers are electors, and some electors who accept bribes are clearly unfit for their duty of electing representatives.

79. Whatever is done skilfully appears to be done with ease; and art, when it is once matured to habit, vanishes from observation. We are therefore more powerfully excited to emulation by those who have attained the highest degree of excellence, and whom therefore we can with the least reason hope to excel. [L.]

80. It is absurd to maintain that when we cannot avoid thinking or conceiving a thing, it must be true; for some persons cannot be in darkness without thinking of ghosts, in which they do not believe. [R.]

81. How can any one maintain that pain is always an evil, who admits that remorse involves pain, and yet may sometimes be a real good? [C.]

82. The time is past in which the transmission of news can be measured by the speed of animals or even of steam; for the telegraph is not approached by either.

[DE MORGAN.]

83. We enjoy a greater degree of political liberty than any civilised people on earth, and can therefore have no excuse for a seditious disposition.

84. Those only who understand other languages are competent to treat correctly of the principles of their own;

since such a competency requires a philosophical view of the nature of language in general. [L.]

85. If matter must be merely phenomenal, I must be so too. [E.]

86. A miracle is incredible, because it contradicts the laws of nature. [E.]

87. There are no practical principles wherein all men agree, and therefore none which are innate. [E.]

88. Potash contains a metal; for all alkalies contain a metal, and potash is an alkali. [E.]

89. Quench not hope; for when hope dies, all dies.

90. That is too bad: you have the impudence to say you are a materialist, while I know that you are a dancing master.

91. Blood is a colour; for it is red, and red is a colour.

92. Every incident in this story is very natural and probable; therefore the story itself is natural and probable.

93. Dolor, si longus, levis; si gravis, brevis: ergo, omnino fortiter sustinendus. [L.]

94. 'Whether we live, we live unto the Lord; and whether we die, we die unto the Lord: whether we live therefore, or die, we are the Lord's.' [*Rom.* xiv. 8.]

95. Quand on n'a point d'amis, on n'est pas heureux; les hommes faux et trompeurs n'ont point d'amis; ainsi les hommes faux et trompeurs ne sont pas heureux.

96. Philippi was the city where the first Christian Church in Europe was founded: it was also the place where the republican army of Rome under Brutus and Cassius was finally defeated; hence the republican army was finally defeated at the city where the first Christian Church in Europe was founded.

97. Switzerland is a republic, and, you will grant, a more stable power is not to be found; nor, again, is any political

society more settled than that of the United States. Surely, then, republican France can be in no danger of revolution. [R.]

98. Expand into a syllogism, as briefly as you can, the argument contained in the dialogue of Shakespeare's King Henry VI. Part III. Act i. Scene 1, between the words, 'my title's good,' etc., and, 'succeed and reign.' [O.]

99. Quoniam deos beatissimos esse constat; beatus autem sine virtute nemo potest: nec virtus sine ratione constare; nec ratio usquam inesse nisi in hominis figura; hominis esse specie deos confitendum est. [H.]

100. Without order there is no living in public society, because the want thereof is the mother of confusion, whereupon division of necessity followeth; and out of division, destruction. [HOOKER, *Ecclesiastical Polity*, v. 8, s. 1.]

101. Whatever is contradictory to universal and invariable experience is antecedently incredible; and as that sequence of facts which is called the order of nature is established, and in accordance with universal experience, miracles or alleged violations of that order are antecedently improbable. [E.]

102. Justice is the profit of others; therefore it is unprofitable to the just man to be just. [O.]

103. In trade both buyer and seller profit: in the home trade both these profits remain in the country; in the foreign trade one profit goes to the foreign trader; therefore the same population will be more profitably employed in the home than in the foreign trade. [O.]

104. Whatever brings in money enriches. Hence the value of any branch of trade, or of the trade of the country altogether, consists in the balance of money it brings in; and any trade which carries more money out of the country than it draws into it is a losing trade: and therefore, money

should be attracted into the country, and kept there by bounties and prohibitions.

105. Distinction may be reasonably expected, because what is not uncommon may be reasonably expected, and distinction is not uncommon. [E.]

106. 'Neither am I moved with envy; for if you are equal to, or less than myself, I have no cause for it; and if you be greater, I ought to endeavour to equal you, and not to speak evil of you.' [L.]

107. Great men have been derided, and I am derided: which proves that my system ought to be adopted.

[DE MORGAN, *Paradoxes*, p. 387.]

108. Preventive measures are always invidious, for when most successful the necessity for them is the least apparent.

109. Treason never prospers: What's the reason? Why, when it prospers, none dare call it Treason.

110. Neque quies gentium sine armis, neque arma sine stipendiis; neque stipendia sine tributis habere queunt.

[TACITUS, *Hist. Lib.* iv. cap. 74.]

111. Men are not brutes; brutes are irrational: all irrational beings are irresponsible; therefore, men are not free from responsibility. [H.]

112. The best of all taxes are taxes on consumption and taxes on the transfer of property: now all the latter and many of the former are levied by stamps; stamp duties therefore are good taxes, and taxes on justice are all stamp duties; therefore taxes on justice are good taxes. (See BENTHAM'S *Protest against Law Taxes*, second edition, 1816, pp. 53, 54.)

113. Dr. Johnson remarked that 'a man who sold a penknife was not necessarily an ironmonger.' What is the name and nature of the logical fallacy against which this remark was directed?

114. When Columbus made the egg stand on end by breaking it, what fallacy may he be said to have committed?

115. 'Either all things are ordered by an intelligent Being who makes the world but one family (and if so, why should a part, or single member complain of that which is designed for the benefit of the whole?); or else we are under the misrule of *atomes*, and confusion. Now, take the case which way you please, there's either no reason or no remedy for complaint; and therefore it is to no purpose to be uneasie.' [Marcus Antoninus' Meditations, ix. 40.]

116. Silk is dearer than wool, and wool than cotton; therefore silk is dearer than cotton.

117. One napoleon is worth twenty francs, and twenty francs are worth about sixteen shillings; therefore one napoleon is worth about sixteen shillings.

118. The Prince of Wales is the eldest son of the reigning sovereign; and the eldest son of the reigning sovereign, if there be such, is the heir to the throne; therefore the Prince of Wales is heir to the throne.

119. It is a mistake to suppose that the rents paid to landlords are a burden on the public, since corn would not be more plentiful or cheaper if they were abolished.

120. The Greeks have little respect for the petty honesty of small tradesmen; we do not greatly admire the wiles of Ulysses; therefore any common inward standard of morals is impossible.

121. Dissent always weakens religion in the people; for it sets itself in opposition to the National Church.

122. 'We are not inclined to ascribe much practical value to that analysis of the inductive method which Bacon has given in the second book of the *Novum Organum*. It is indeed an elaborate and correct analysis. But it is an analysis of that which we are all doing from morning to

night, and which we continue to do even in our dreams.'—MACAULAY, *Essay on Bacon*. [E.]

123. The Claimant has undoubtedly many peculiarities of gait and manner which were characteristic of the missing baronet. Are not these therefore proofs of identity equivalent to the evidence of imposture afforded by the absence of tattoo-marks which the genuine man is proved to have possessed?

124. Even if it could be shown that animals perform certain actions which men could only perform by the aid of reason, it would by no means necessarily follow that animals perform them by its aid. [C.]

125. If there's neither Mind nor Matter,
Mill's existence, too, we shatter:
If you still believe in Mill,
Believe as well in Mind and Matter. [E.]

126. If we accept Aristotle's testimony, we may infer that Anaximander was not one of the Ionian philosophers that accepted as the One material principle a mean term between Water and Air; for, in the *Physics*, we read that he held the substances in nature to have been produced from the primordial principle by a process of secretion and not by a process of condensation and rarefaction; while in the *De Coelo* it is stated that other mode of production than the last-named was not put forward by any who adopted such a mean term for their principle. What syllogistic form (figure and mood) does this inference most naturally assume? [R.]

CHAPTER XXI

ELEMENTS OF EQUATIONAL LOGIC

1. THE symbols employed in this system are the following:—

A, B, C, or other capital letters, signify qualities, or groups of qualities, forming the common part, or intensive meaning, of terms, or names of objects and classes of objects.

a, *b*, *c*, or other small italic letters, are the corresponding negative terms; thus *a* signifies the absence of one or more of the qualities signified by A. This notation for negatives was proposed by De Morgan (*Formal Logic*, p. 38). The mark = is the sign of Identity of Meaning of the terms between which it stands; thus A = B indicates that the qualities signified by A are identical with the qualities signified by B.

The sign ·|· signifies *unexclusive alternation*, including the ordinary meanings of both the conjunctions *or* and *and*. Thus A ·|· B means the qualities of A or those of B, or those of both A and B, if they happen to coincide.

Juxtaposition of two letters forms a term whose meaning is the sum of the qualities signified by the two letters: thus AB means a union of the qualities of A and B.

2. The Laws of Combination of these symbols are as follow:—

The Law of Commutation. AB = BA: that is to say, the sum of qualities of A and B is evidently the same as the sum of qualities of B and A. The way of arriving at the sum may be different, but the result is identical.

The Law of Simplicity. AA = A: if we have the same qualities twice over we get the same as if we named them once.

The Law of Unity. A ·|· A = A: the qualities of A $\frac{\text{and}}{\text{or}}$ the qualities of A are simply the qualities of A.

The Law of Distribution. A(B ·|· C) = AB ·|· AC. The qualities of A with those of B $\frac{\text{and}}{\text{or}}$ those of C are the same as those of AB $\frac{\text{and}}{\text{or}}$ those of AC.

The Law of Indifferent Order. B ·|· C = C ·|· B, which is sufficiently evident.

3. The Laws of Thought are the foundation of all reasoning, and may thus be symbolically stated:—

The Law of Identity	A = A.
The Law of Duality or of Excluded Middle . . .	A = AB ·|· A*b*.
The Law of Contradiction .	A*a* = 0.

The successive application of the Law of Duality to two, three, four, five or more terms, gives rise to the development of all possible logical combinations, called the Logical Alphabet, the first few columns of which are given below. The combinations for six terms are given in the *Principles of Science*, p. 94 (first ed. vol. i. p. 109).

The Logical Alphabet

I.	II.	III.	IV.	V.	VI.	VI.—*continued.*
X	AX	AB	ABC	ABCD	ABCDE	aBCDE
	aX	Ab	ABc	ABCd	ABCDe	aBCDe
		aB	AbC	ABcD	ABCdE	aBCdE
		ab	Abc	ABcd	ABCde	aBCde
			aBC	AbCD	ABcDE	aBcDE
			aBc	AbCd	ABcDe	aBcDe
			abC	AbcD	ABcdE	aBcdE
			abc	Abcd	ABcde	aBcde
				aBCD	AbCDE	abCDE
				aBCd	AbCDe	abCDe
				aBcD	AbCdE	abCdE
				aBcd	AbCde	abCde
				abCD	AbcDE	abcDE
				abCd	AbcDe	abcDe
				abcD	AbcdE	abcdE
				abcd	Abcde	abcde

4. The one sole and all sufficient rule of inference is the following Rule of Substitution.

For any term substitute what is stated in any premise to be identical in meaning with that term.

The term may consist of any single letter, any juxtaposed letters, or any group of alternatives connected by the sign ·|·, the sign of unexclusive alternation.

5. It is assumed as a necessary law that every term must have its negative. This was called the *Law of Infinity* in my first logical essay (*Pure Logic*, p. 65 ; see also p. 45) ; but as pointed out by Mr. A. J. Ellis, it is assumed by De Morgan, in his *Syllabus*, article 16. Thence arises what I propose to call the Criterion of Consistency, stated as follows :—

Any two or more propositions are contradictory when, and only when, after all possible substitutions are made, they occasion the total disappearance of any term, positive or negative, from the Logical Alphabet.

The principle of this criterion was explained in p. 65 of the Essay on *Pure Logic* referred to, but subsequent inquiry, and the writings of Mr. A. J. Ellis, have tended to show the supreme importance of the criterion.

The processes of this equational system of Logic are fully treated in the first seven chapters of the *Principles of Science*, and they are now amply illustrated by the problems which follow.

6. How do you express in the new logic the four Aristotelian forms of proposition indicated by the vowels **A**, **E**, **I**, and **O**?

The answer is—

A.	Every *A* is *B*.	A = AB	(1).
E.	No *A* is *B*.	A = A*b*	(2).
I.	Some *A* is *B*.	CA = CAB	(3).
O.	Some *A* is not *B*.	CA = CA*b*	(4).

The first expresses the coincidence of the class A with part of the class B, namely A B, which is the equational mode of asserting that the A's form part of the B's. The second expresses similarly that the A's are found among the not-B's. In the third form *some* is expressed by the symbol C, and the proposition asserts that some A's (CA) are identical with a part of the class B. Some difficulties may arise about this form, owing to the ambiguity of the Aristotelian *some*, as elsewhere discussed (pp. 151–158). The fourth proposition is evidently the negative form of the third.

7. How shall we express equationally the assertion of Hobbes (*De Corpore Politico*, I. i. 13), that 'Irresistible might in the state of nature is right'?

'Might' is the principal part of the subject, but it is qualified or restricted in this proposition by the adjective 'irresistible,' and by the adverbial 'in the state of nature.' Thus putting

A = irresistible; C = in the state of nature;
B = might; D = right.

The subject is clearly A B C; and D is affirmed of it. But Hobbes cannot, of course, have meant that all right is irresistible might; only in the state of nature is this true. As, indeed, irresistible might must overcome everything opposed to it, there can be nothing else right in the sphere of its action, so that the proposition would seem to have the form A B C = C D. It is not easy to be sure of the meaning even of Hobbes.

8. Represent the meaning of the sentence 'Man that is born of a woman, is of few days, and full of trouble.'

The relative clause, 'that is born of a woman,' is evidently explicative, and we cannot suppose that there are any men not 'born of a woman.' Hence taking

A = man; C = of few days;
B = born of a woman; D = full of trouble;

the meaning seems to be expressed in the two propositions—

$$A = AB;$$
$$A = ACD.$$

But if we may treat the sentence as an imperfectly expressed syllogism—namely, 'because man is born of a woman he is, etc.'—then the premises obviously become A = AB, and B = B C D, and the conclusion by substitution for B in the first of its value in the second, is A = ABCD.

9. Show how to obtain equationally the contrapositive of $A = AB$.

This is explained in the *Principles of Science*, p. 83 (first edition, vol. i. pp. 97-102), but may be thus briefly repeated—

By the Second Law of Thought

$$b = Ab \cdot|\cdot ab.$$

Substitute for A its equal AB.

$$b = ABb \cdot|\cdot ab = 0 \cdot|\cdot ab,$$

$$\text{or, } b = ab.$$

Concerning the contrapositive see above, pp. 32, 43-47, etc.

10. Show how to obtain the complete contrapositive of $A = B$.

As before

$$b = Ab \cdot|\cdot ab = Bb \cdot|\cdot ab = 0 \cdot|\cdot ab = ab;$$

similarly

$$a = aB \cdot|\cdot ab = aA \cdot|\cdot ab = 0 \cdot|\cdot ab = ab.$$

Having now the two propositions

$$a = ab = b,$$

it is plain that we may eliminate ab, and get

$$a = b.$$

11. What descriptions of the terms 'glittering thing' and 'not gold' can you draw from the following assertions?—

(1) Brass is not gold;
(2) Brass glitters.

Let A = brass; B = golden;
C = glittering thing.

The premises are

$$(1) \quad A = Ab; \quad (2) \quad A = AC.$$

Obviously $C = ABC \cdot|\cdot AbC \cdot|\cdot aBC \cdot|\cdot abC$.

The first of the alternatives ABC is negatived by (1); but the second and fourth coalesce, and we have

$$C = bC \cdot|\cdot aBC;$$

that is, a glittering thing is either not golden, or else it is golden, and then not brass.

For b we similarly get

$$b = bC \cdot|\cdot abc.$$

Show that we may also infer

$$C = C\ (a \cdot|\cdot b),$$
$$\text{and } b = b\ (a \cdot|\cdot C).$$

12. How shall we represent in the forms of Equational Logic the moods of the old syllogism?

All of the moods without exception may be solved by the *indirect method*, that is by working out the combinations consistent with the premises. Most of the moods may, however, be solved also by direct substitution, as will be seen in the following examples:—

Barbara.

All men are mortal;	(1)	$B = BC$.
All kings are men;	(2)	$A = AB$.
∴ All kings are mortal;	(3)	$A = ABC$.

We get (3) by substituting for B in (2), its equivalent BC in (1). The conclusion amounts to saying that 'king' is equivalent to 'king—man—mortal.' If desired, we can by further substitution of A for AB in (3) obtain $A = AC$, or king = king—mortal, which is a precise expression for the Aristotelian conclusion 'All kings are men.'

Celarent

No men are perfect;	(1)	$B = Bc$.
All kings are men;	(2)	$A = AB$.
No kings are perfect;	(3)	$A = ABc$.

Solved, as in the last case, by direct substitution in (2) of the value of B given in (1).

Darii.

All mathematicians have well-trained intellects;	$C = CD$.
Some women are mathematicians;	$AB = ABC$.
Some women have well-trained intellects.	$AB = ABCD$.

Here A stands for the indefinite adjective *some*, and B for women, and we then treat AB as an undivided term, and obtain the result by direct substitution, exactly as in the previous moods.

Ferio.

No foraminifera are fresh-water inhabitants;	$C = Cd$.
Some components of chalk are foraminifera;	$AB = ABC$.
Some components of chalk are not fresh-water inhabitants.	$AB = ABCd$.

Except that a negative term *d* takes the place of the positive term D, in the last mood, there is no difference in form between them. In fact, all the four moods of the first figure present so great similarity that they may be said to be of one form of inference.

Cesare.

The absolute is not phenomenal;	(1)	$C = Cb$.
All known things are phenomenal;	(2)	$A = AB$.
All known things are not the absolute.	(3)	$A = ABc$.

We cannot by any direct substitution obtain the conclusion from the premises, as B appears in (2) and b in (1). But we may take the contrapositive of (1) as described before (p. 184), namely, $B = Bc$, and substitution in the second premise is then practicable.

Camestres.

All laws of nature are invariable;	(1)	$C = CB$.
No human customs are invariable;	(2)	$A = Ab$.
No human customs are laws of nature.	(3)	$A = Ac$.

As in the last mood, we must take the contrapositive of (1), namely $b = bc$, and substitute thereby in (2).

13. The equational treatment of the moods Camestres, Cesare, and Camenes is described also in the *Principles of Science*, pp. 84-86 (first edition, vol. i. pp. 99-101), or in the *Substitution of Similars*, pp. 47-49; but the following is the briefest way of getting the Aristotelian conclusion, of Camestres, as suggested by Mr. W. H. Brewer, M.A.

Let the premises be

(1) $A = AC$.
(2) $B = Bc$.

Multiply together, and we get

$$AB = ABCc = 0$$

Thus there is no such thing as AB.

Similarly with Camenes.

14. The remaining moods need only be symbolically represented. In every case D = some:

	Major Premise.	Minor Premise.	Conclusion.
Festino . .	A = Ab .	CD = BCD .	CD = aBCD
Baroko . .	A = AB .	CD = bCD .	CD = abCD
Darapti . .	B = AB .	B = CB .	AB = CB
Disamis . .	BD = ABD .	B = BC .	BD = ABCD
Datisi . . .	B = AB .	BD = BCD .	BCD = ABD
Felapton . .	B = aB .	B = BC .	BC = aB
Bokardo . .	BD = aBD .	B = BC .	BCD = aBCD
Ferison . .	B = aB .	BD = BCD .	BCD = aBD
Bramantip .	A = AB .	B = BC .	ABC = A
Camenes . .	A = AB .	B = Bc .	C = aC
Dimaris . .	AD = ABD .	B = BC .	AD = ABCD
Fesapo . .	A = Ab .	B = BC .	BC = aBC
Fresison . .	A = Ab .	BD = BCD .	BCD = aBCD

15. Exhibit the logical force of the motto adopted by Sir W. Hamilton—

(1) In the world there is nothing great but Man.

(2) In Man there is nothing great but Mind.

Let A = in the world; C = possessing mind;
B = man; D = great.

The conditions may be represented as—

(1) A = ABD ·|· Abd.
(2) B = BCD ·|· Bcd.

As it may be understood, though unexpressed, that

(3) all men are in the world, and that (4) all possessing mind are men, we have further

$$(3)\quad B = AB. \qquad (4)\quad C = CB.$$

The combinations are thus reduced to

$$\begin{array}{ll} ABCD & abcD \\ Abcd & abcd. \end{array}$$

Observe that, if mind were regarded not as an attribute but as a physical part of man, we could not treat the assertion as one of simple logical relation.

16. What is the meaning of the assertion that 'All the wheels which come to Croyland are shod with silver'?

If we take

A = wheel;
B = coming to Croyland;
C = shod with silver,

the assertion, as it stands, is evidently in the form

$$AB = ABC \qquad (1).$$

But it was always understood, no doubt, that this adage was to be joined in the mind with the tacit premise 'No wheels are shod with silver,' expressed by

$$A = Ac \qquad (2).$$

There seems, at first sight, to be contradiction between these premises; for (1) speaks of wheels shod with silver and (2) denies that there are such things. The explanation is obvious, namely, that there are no such things as wheels coming to Croyland. Of the four combinations containing A, ABc is negatived by (1), and ABC and AbC by (2), so that the description of A is given thus,

$$A = Abc,$$

or, by substitution of A for Ac,

$$A = Ab,$$

that is, no wheels come to Croyland. This is of course the inference which the adage was intended to suggest, Croyland being an ancient abbey lying among the fens of Cambridgeshire, where in former days no wheeled vehicle could make its way.

This question illustrates the important logical principle that all propositions ought, strictly speaking, to be interpreted hypothetically. We have only to put these premises in the hypothetical form, and we see that they make a reasonable destructive hypothetical syllogism—thus

If any wheels come to Croyland they are shod with silver; but no wheels are shod with silver; therefore, no wheels do come to Croyland.

17. Ruminant animals are those which have cloven feet, and they usually have horns; the extinct animal which left this footprint had a cloven foot; therefore it was a ruminant animal and had horns. Again, as no beasts of prey are ruminant animals it cannot have been a beast of prey.

The above problem is given in the *Elementary Lessons* (p. 321, No. 78). Taking our symbols thus—

A = ruminant; D = extinct animal;
B = having cloven feet; E = beast of prey;
C = having horns;

we have clearly $A = B$ (1).

The statement that ruminant animals *usually*[1] have horns may be formalised as

$$BA = BAC \qquad (2),$$

that is to say, a certain particular portion of the class A, *BA* have horns. Next we have

$$D = DB \qquad (3).$$

Substituting in (3) by (1) we get

$$D = DA \qquad (5);$$

showing that the extinct animal was a ruminant. But, as we cannot substitute between (3) and (2), it is erroneous to assert that it had horns. If by *usually* we mean in the far greater number of cases, then there is a considerable probability, but no certainty that it had horns.

Again, we have as an additional premise, that beasts of prey are not ruminant, or

$$E = aE \qquad (4),$$

which, taken with

$$D = DA \qquad (5),$$

our previous conclusion, gives a syllogism in Cesare, establishing that D is not E, or that the extinct animal cannot have been a beast of prey. This we might sufficiently prove symbolically by multiplying the respective members of (4) and (5) together, giving

$$DE = aEDA = 0.$$

This shows that inconsistency arises from supposing that this D can be also E. The same result might be worked out by combinations, giving $D = De$.

[1] The first edition of the *Lessons* reads *always* instead of *usually*.

18. Take the proposition 'All crystals are solids,' and ascertain precisely what it affirms, what it denies, and what it leaves doubtful.

[A. J. ELLIS.]

Taking A = crystal, and B = solid, the proposition is in the form

$$A = AB.$$

The conceivable combinations are four in number, namely,

AB, A*b*, *a*B, and *ab*.

Of these, only A*b* is inconsistent with the premise, that is to say, the premise 'All crystals are solids' denies the existence of such things as 'unsolid crystals.' We cannot strike out AB, because then there would be no such thing as crystals left; hence the premise affirms the existence of solid crystals, in the sense that any other proposition denying that crystals are solid, or that solids may be crystals, would stand in contradiction to our premise.

Again, we may not strike out *a b*, because there would then be no such thing as *b*, or not-solids. Hence to avoid contradiction of our premise, there must be such a thing as 'a non-crystal which is not-solid.' If we are to hold to our adopted Criterion of Consistency (p. 181), we must say that at least one case of *ab* exists, so that to avoid self-contradiction, *some*, that is at least one case of not-crystal, must be allowed to exist [1] and to be not-solid. This confirms the conclusion which we previously obtained by the Aristotelian logic from the same premise (p. 48). But the combination *a*B may be removed or left without affecting the truth of the premises, which therefore leave it entirely in doubt whether 'not-crystals which are solids exist.'

[1] Concerning the logical sense of the verb *exist*, see pp. 141-2.

'Not-crystals, not-solids' must exist, but 'Not crystals-solids,' may or may not. But if they do not, then crystals and solids will be coincident classes.

To sum up—

$A = AB$ — affirms that all As are Bs;
$A = AB$ — affirms that all not-Bs are not-As;
$A = AB$ — affirms that some not-As are not-Bs;
$A = AB$ — denies that all not-As are not-Bs;
$A = AB$ — leaves doubtful whether not-A can be B.

19. Are the following propositions equivalent each to the other?—

(1) All who were there talked sense;
(2) All who talked nonsense were away.

[DE MORGAN.]

Putting

A = being there;	a = not being there;
B = talking;	b = not talking;
C = sensible;	c = not sensible;

the first premise seems to mean that all who were there (A), talked and were sensible; that is

$$A = ABC \quad (1).$$

The second is to the effect

$$Bc = aBc \quad (2);$$

that is to say, those who talked (B), but not sensibly (c), were away (a). Now (1) negatives three combinations, ABc, AbC, and Abc; whereas (2) negatives only ABc. They are, therefore, very different propositions; for (2) allows that some may have been present who did not talk at all (b), whether sensible people or not (C or c). Nevertheless, there is this much relation between the two propositions, that we can infer (2) from (1). If all who were there talked sense, those people who talked nonsense, assuming there to be such persons, must have been absent. But we cannot invert this relation. Because those who talked nonsense were away, it does not follow that those who were present talked sense; they may all have been silent.

20. De Morgan says (*Syllabus*, p. 14), 'Any one who wishes to test himself and his friends upon the question whether analysis of the forms of enunciation would be useful or not, may try himself and them on the following question:—
Is either of the following propositions true, and if either, which?
(1) All Englishmen who do not take snuff are to be found among Europeans who do not use tobacco.
(2) All Englishmen who do not use tobacco are to be found among Europeans who do not take snuff.
Required—immediate answer and demonstration.'

Assigning symbols as follows:—

A = Englishmen; C = taking snuff;
B = Europeans; D = using tobacco;

it is pretty obvious that the above propositions are thus symbolised—

$$(1) \quad Ac = ABcd.$$

$$(2) \quad Ad = ABcd.$$

We are to compare these with the well-known relations of the terms, which may be assumed to be

$$(3) \quad A = AB;$$

that is, 'Every Englishman is an European,' and

$$(4) \quad C = CD;$$

that is, 'All who take snuff use tobacco.' Now, in working out the combinations, we find that the class Ac is composed under conditions (3) and (4) as follows:

$$Ac = ABcD \cdot|\cdot ABcd.$$

The truth is, then, that Englishmen who do not take snuff consist of English Europeans not taking snuff, but using tobacco, and of English Europeans neither taking snuff nor using tobacco. In short (1) is erroneous in ignoring the fact that some Englishmen not using snuff may be Europeans who do use tobacco for smoking.

According to assumption (2) the description of Ad is $ABcd$, which coincides with the description drawn from (3) and (4). Thus it is true that all Englishmen who do not use tobacco are to be found among Europeans who do not take snuff; the negation of the larger term, using tobacco, includes the negation of the narrower one, using snuff. But it by no means follows that because our inference about Ad is the same from (2) as from (3) and (4), therefore these conditions are identical, as will be seen in the following descriptions of the class A as furnished under the several suppositions and conditions—

(1) $A = ABC \cdot|\cdot ABcd \cdot|\cdot AbC.$

(2) $A = ABCD \cdot|\cdot ABc \cdot|\cdot AbD.$

(3) and (4) $A = ABCD \cdot|\cdot ABc.$

21. What can we infer about the term Europeans from the following premises?—

(1) All Continentals are Europeans;

(2) All English are Europeans;

(3) No English are Continentals.

Taking A, B, C to represent Continentals, English, and Europeans respectively, the premises become

$$(1)\ A = AC.$$
$$(2)\ B = BC.$$
$$(3)\ B = aB.$$

The combinations left uncontradicted are the four—A*b*C, *a*BC, *ab*C, *abc*, whence we learn that Europeans, C, consist of Continentals who are not English, of English who are not Continentals, and of any others, who are neither Continentals nor English (*ab*C).

22. Criticise Thomson's 'Immediate inference by the sum of several predicates. . . . From a sufficient number of judgments in **A**, having the same subject, a judgment in **U** may be inferred, whose predicate is the sum of all the other predicates.' [P.]

This question has been answered in the *Principles of Science*, p. 61 (first ed. Vol. I. p. 73). Judgments in **A** are of the form $P = PQ$, $P = PR$, $P = PS$, etc., and by summing up the predicates by successive substitution in the second side of $P = PQ$, we may get $P = PQRS$ . . . . But this does not give a proposition of the form **U** which, as described by Thomson, is of the form $P = X$.

23. Represent the following argument from Thomson's *Laws of Thought*, § 107 :—

All P is either C or D or E ;
S is neither C nor D nor E ;

therefore, S is not P.

The premises are respectively :—

$$P = PC \cdot|\cdot PD \cdot|\cdot PE.$$
$$S = Scde.$$

We get the conclusion in the briefest way by multiplying the two premises together as they stand ; thus :—

$$PS = P\ (C \cdot|\cdot D \cdot|\cdot E)\ Scde = 0 \cdot|\cdot 0 \cdot|\cdot 0.$$

Each alternative is found to be contradictory, so that there is no such thing as PS, that is to say, no P is S.

The argument is not, however, correctly described by Dr. Thomson as in the syllogistic mood **UEE**, nor are the other forms of argument given in the same section syllogistic. They are disjunctive in character.

24. If Abraham were justified, it must have been either by faith or by works: now he was not justified by faith (according to James), nor by works (according to Paul): therefore Abraham was not justified. [W.]

There is some difficulty in deciding on the best method of symbolising this argument, owing to the vagueness of the conditions when analysed; but the following seems to be the best representation :—

Let A = Abraham ; C = justified by works ;
B = justified ; D = justified by faith.

Then the premises are :—

$$AB = AB\ (C \cdot|\cdot D).$$
$$A = Ac.$$
$$A = Ad.$$

These premises will be found to erase all the combinations of A excepting A*bcd*, which gives the conclusion. The combinations of *a* are altogether unaffected and need not be examined. The student may try other modes of representing the premises, but should get $A = Ab$ by every method.

25. It must be admitted, indeed, that (1) a man who has been accustomed to enjoy liberty cannot be happy in the condition of a slave: (3) many of the negroes, however, may be happy in the condition of slaves, because (2) they have never been accustomed to enjoy liberty. [W.]

Let A = man accustomed to enjoy liberty;
B = happy in condition of slave;
C = certain negroes.

The premises may be stated in the forms

$$A = Ab. \quad (1)$$
$$C = aC. \quad (2)$$

The supposed conclusion is $C = CB$. (3)

The possible combinations as in the margin, from which it will be seen that

$$C = aBC \cdot|\cdot abC;$$

that is, are either B, happy, or *b*, not happy.

A*bc*
*a*BC
*a*B*c*
*ab*C
abc

The fallacy is that of Negative Premises or of Illicit Process of the Major.

26. If that which is devoid of heat and devoid of visible motion is devoid of energy, it follows that what is devoid of visible motion but possesses energy cannot be devoid of heat.

Let $A =$ possessing heat;
$B =$ possessing visible motion,
$C =$ possessing energy,

the universe being 'things' unexpressed, and 'devoid of' being taken as the negative of 'possessing.' Then the condition is:—

$$ab = abc.$$

By contraposition we obtain, using Mr. MacColl's notation for the negative of ab (*See* Preface):

$$C = C\ (ab)'$$
$$= C\ (Ab \cdot|\cdot aB \cdot|\cdot AB)$$

Hence $\quad bC = AbC,$

two self-contradictory alternatives disappearing. It can also be readily shown that this inference is equivalent to the original condition.

27. Prove the logical equivalence of the proposition $B = AC \cdot|\cdot ac$ and $b = Ac \cdot|\cdot aC$.

This might be shown by receding to the combinations of the Logical Alphabet, but it is more neatly proved by equating the negatives of each member of the first equation. If $M = N$, then also $m = n$ (p. 184); hence the negative of B must be identical with the negative of $AC \cdot|\cdot ac$. Now the negative of B is b; that of the compound and complex member is the compound of the negatives of the two alternatives. In Mr. MacColl's notation

$$(AC \cdot|\cdot ac)' = (AC)'\ (ac)'$$
$$= (aC \cdot|\cdot Ac \cdot|\cdot ac)\ (Ac \cdot|\cdot aC \cdot|\cdot AC).$$

On multiplying out, the nine products are found to be all self-contradictory excepting $aC \cdot|\cdot Ac$, which is therefore the expression for b. *Vice versâ* the negative of $Ac \cdot|\cdot aC$ will be found to be $AC \cdot|\cdot ac$, so that the propositions are clearly equivalent.

28. If no A is BC, what can I infer about the relation of B and AC?

The condition is

$$A = Ab \cdot|\cdot Ac.$$

Substitute in either side of

$$ABC = ABC,$$

and we get

$$ABC = ABbC \cdot|\cdot ABCc = 0,$$

or B cannot be AC.

29. It is known of certain things that (1) where the quality A is, B is not; (2) where B is, and only where B is, C and D are. Derive from these conditions a description of the class of things in which A is not present, but C is.

The premises are clearly:—

$$(1)\ .\ .\ A = Ab;$$
$$(2)\ .\ .\ B = CD$$

The conceivable constituents of the class which is C but not-A are—

$$aC = aBCD \cdot|\cdot aBCd \cdot|\cdot abCD \cdot|\cdot abCd.$$

Substituting CD for B in the second, and B for CD in the third alternative, we find that these combinations give contradictory results, namely $aBCDd$ and $abBCD$. It follows that

$$aC = aBCD \cdot|\cdot abCd.$$

Observe that the premise (1) has no connection with this result, which is deducible from (2) alone.

30. It has been observed that in a certain class of substances, (1) where the properties A and B are present, the property C is present; and (2) where B and C are present, A is present. Does it follow that B is present where C and A are present?

The premises are obviously:—

$$(1) \quad . \quad . \quad AB = ABC,$$
$$(2) \quad . \quad . \quad BC = ABC,$$

which are equivalent to the single proposition

$$AB = BC.$$

The answer is obtained at once from the combinations in the margin, which show that

$$AC = ABC \cdot|\cdot AbC;$$

that is, in the presence of A and C, B is indifferently present or absent. [Dr. Macfarlane (*Algebra of Logic*, 1879, p. 141), who gives this

ABC
AbC
Abc
aBc
abC
abc

problem, requires more than half a page to solve it, using, moreover, sundry impossible logical fractions.]

31. Given (1) that everything is either B or C, and (2) that all C is B, unless it is not A: prove that all A is B. [C.]

The first condition is expressed by the assertion that every not-B is C, which carries with it the equivalent that every not-C is B. Thus we have

$$b = bC. \quad (1)$$

The second assertion is less easy to interpret, because we are not told what happens if 'it,' that is C, is not-A. The meaning appears, however, to be that C if it is A must be B, that is—

$$AC = ABC. \quad (2)$$

These conditions give the combinations—

ABC.	*a*BC.
AB*c*.	*a*B*c*.
	*ab*C.

from the first two of which we learn that A is always B.

32. If we throw 'every A is B' into the form 'every A is B or B,' we have 'every A which is not B is B'—a contradiction in terms. But it evidently implies that there can be no As which are not Bs, and thus we return to 'every A is B.'

The above is a transcript with altered symbols from De Morgan's sixth example (*Formal Logic*, p. 123). But the contradiction arises simply from an error in not multiplying both alternatives by b. De Morgan follows the rule for the resolution of dilemmas, not observing that this rule can apply only when the alternatives are different. Equationally we have—

$$A = AB \cdot|\cdot AB.$$

hence . . . $Ab = ABb \cdot|\cdot ABb = 0.$

he gets . . . $Ab = ABb \cdot|\cdot AB = AB.$

It is rarely we find De Morgan tripping.

33. Every A is one only of the two B or C; D is both B and C, except when B is E, and then it is neither; therefore no A is D.

This problem was proposed by De Morgan in the *Formal Logic*, p. 124, and a solution has been given in the *Principles of Science*, p. 101 (first ed., vol. i. p. 117). The premises, as Professor Croom Robertson has pointed out, may be stated in two propositions, namely—

$$A = ABc \cdot|\cdot AbC.$$
$$D = DeBC \cdot|\cdot DEbc.$$

Some objection has, however, been taken by Mr. Monro to my solution, and the student will find a good exercise in going over the solution carefully. It seems rather doubtful how we should treat the combinations which are E and not B; but the difficulties lie wholly in the interpretation of De Morgan's conditions.

34. From A follows B, and from C follows D; but B and D are inconsistent with each other. Hence A and C are inconsistent with each other.

This problem, which is formally the same as one of De Morgan's (*Formal Logic*, p. 123, Example 3), has the conditions

$$(1)\ A = AB;$$
$$(2)\ C = CD;$$
$$(3)\ B = Bd.$$

The consistent combinations are

AB*cd*.	*ab*CD.
*a*B*cd*.	*abc*D.
	abcd.

We see that A and C never occur together, and in fact that A is never found excepting in the presence of B and the absence of both C and D.

I committed an error in treating this problem in the *Substitution of Similars* (pp. 52, 53), by regarding *a*B*cd* as negatived by the premises. B may occur in the absence of A, but C and D must both be absent.

35. What are the combinations of the qualities A, B and C which are possible according to the following conditions? (1) 'Where A is present, B and C are either both present at once or absent at once; (2) where C is present, A is present.' Describe the class not-B under these conditions.

The conditions are expressed equationally as

$$(1)\ A = ABC \cdot|\cdot Abc.$$
$$(2)\ C = AC.$$

ABC
A*bc*
*a*B*c*
abc

The consistent combinations are shown in the margin; AB*c* and A*b*C are removed by the first condition, and *a*BC and *ab*C by the second. Selecting the two remaining ones which contain *b*, we have the required description—

$$b = Abc \cdot|\cdot abc = bc.$$

Where B is absent, C also will be absent.

36. 'The logical value of two affirmative premises in the second figure is absolute zero.' Examine the truth of this statement. [P.]

The two premises assumed to be universal may be symbolised as

$$(1)\ A = AB;$$
$$(2)\ C = CB.$$

The first negatives the combinations A*b*C and A*bc*, the second A*b*C and *ab*C, so that the premises overlap in regard to A*b*C. There remain five combinations. If we inquire what is A we get the value

$$A = ABC \cdot|\cdot ABc = AB,$$

which is no more than (1). For the description of C similarly we get (2). Thus it is plain that no relation is established between A and C. Concerning B we have even less information; for

$$B = ABC \cdot|\cdot ABc \cdot|\cdot aBC \cdot|\cdot aBc = AB \cdot|\cdot aB = B.$$

Of the negative terms, however, we draw more significant descriptions; thus

$$a = a\mathrm{BC} \cdot|\cdot ac.$$
$$b = abc.$$
$$c = \mathrm{B}c \cdot|\cdot abc.$$

It cannot be truly said that the logical value of the premises is absolute zero.

37. Given that (1) everything which is B but not D is either both A and C or neither A nor C; and (2) that neither C nor D is both A and B: prove that no A is B.

[Adapted from Moral Science Tripos, Cambridge, 1879.]

The conditions are

$$(1)\ \mathrm{B}d = \mathrm{B}d\,(\mathrm{AC} \cdot|\cdot ac).$$
$$(2)\ \mathrm{C} \cdot|\cdot \mathrm{D} = (\mathrm{C} \cdot|\cdot \mathrm{D})\,(a \cdot|\cdot b).$$

Confining our attention to the combinations containing AB, we see that ABcd is contradicted by (1), and the rest which contain either C or D, by (2). Hence there are no ABs, or no A is B.

The equation (2) may be more briefly stated as

$$\mathrm{AB} = \mathrm{AB}cd.$$

The only combination containing a removed by (1) and (2) is aBCd.

38. Illustrate the use of symbolic methods by expressing the propositions—

(1) No A is B except what is both C and D, and only some of that.

(2) Either C or D is never absent except where A or B is present, but both are always absent then. [C.]

The first proposition appears to deny the presence of any combination containing AB except there be also present C and D, and only in some cases then. To express this *some* we must introduce another letter term, say E, so that where E is present the above holds true; where E is absent, A is not B at all. We find then that the following combinations are negatived:

$ABCdE$.	$ABCDe$.
$ABcDE$.	$ABCde$.
$ABcdE$.	$ABcDe$.
	$ABcde$.

All this may be expressed in the one equation

$$AB = ABCDE.$$

The proposition (2) is not easy to interpret, but seems to mean

$$A \cdot|\cdot B = cd.$$

39. Every X is either P, Q, or R; but every P is M, every Q is M, every R is M; therefore every X is M.

De Morgan, who gives the above (*Formal Logic*, p. 123, Example 5), describes it as a common form of the dilemma. It is thus solved equationally:

$$\begin{aligned}
&(1)\quad X = X\,(P \cdot|\cdot Q \cdot|\cdot R);\\
&(2)\quad P = PM;\\
&(3)\quad Q = QM;\\
&(4)\quad R = RM.
\end{aligned}$$

Substituting by (2) (3) (4) in (1),

$$\begin{aligned}
X &= X\,(PM \cdot|\cdot QM \cdot|\cdot RM);\\
X &= X\,(P \cdot|\cdot Q \cdot|\cdot R)\,M.
\end{aligned}$$

Re-substituting in the last by (1)—

$$X = XM.$$

40. Every A is either B, C, or D; no B is A; no C is A; therefore every A is D.
[De Morgan, *Formal Logic*, p. 122.]

The premises are clearly—

$$\begin{aligned}
&(1)\quad A = AB \cdot|\cdot AC \cdot|\cdot AD.\\
&(2)\quad B = aB.\\
&(3)\quad C = aC.
\end{aligned}$$

In (1) substitute the values of B and C given in (2) and (3), and then strike out two self-contradictory terms—

$$A = AaB \cdot|\cdot AaC \cdot|\cdot AD = AD.$$

41. If A be B, E is F; and if C be D, E is F; but either A is B, or C is D; therefore, E is F. (De Morgan, *Formal Logic*, p. 123.)

This appears to be more complicated in symbols than it really is. The first two premises are—

$$(1) \quad AB = ABEF$$
$$(2) \quad CD = CDEF.$$

To express the third premise we must introduce explicitly the tacit term, say X, meaning the *circumstances* under which the proposition holds good,—in this place, or at this time, or under certain assumed conditions. Thus we have—

$$(3) \quad X = XAB \cdot|\cdot XCD.$$

substituting by means of (1) and (2),

$$X = (XAB \cdot|\cdot XCD)EF,$$

and re-substituting by (3)

$$X = XEF.$$

42. 'Every A is either B or C, and every C is A.' This, says De Morgan (*ibid.* p. 123), is wholly inconclusive, and leads to an identical result.

Equationally treated this is not quite so. The premises are—

$$(1) \quad A = AB \cdot|\cdot AC$$
$$(2) \quad C = AC$$
$$\text{Hence } (3) \quad A = AB \cdot|\cdot C.$$

De Morgan finds that A*b* is C, which C being A gives A*b* is A a necessary proposition or truism. But we also get, multiplying each side of (3) by *b*,

$$Ab = ABb \cdot|\cdot bC = bC.$$

In the absence, then, of B, there is identity between A and C, but in the presence of B, A may be either B or C.

43. Every A is B or C or D; every B is E; every C is E; and every E is D.

[De Morgan, *ibid.* p. 123, Example 4.]

Thus symbolised—

$$(1) \quad A = AB \cdot|\cdot AC \cdot|\cdot AD.$$

$$(2) \quad B = BE. \qquad (3) \quad C = CE. \qquad (4) \quad E = ED.$$

By obvious substitutions, by (2) and (3) in (1), and then by (4) in the result, we get—

$$A = ABDE \div ACDE \div AD.$$

But the first two of these alternatives are superfluous; they both involve D and are therefore contained in the wider term AD. Hence—

$$A = AD.$$

44. 'If the relations A and B combine into C, it is clear that A without C following means that there is not B, and that B without C following means that there is not A.'

[De Morgan, *Third Memoir on the Syllogism*, p. 48.]

The relations A and B combining into C appears to mean simply that AB is accompanied by C, or—

$$AB = ABC.$$

To find A without C following, we have necessarily

$$Ac = ABc \cdot|\cdot Abc.$$

Inserting for AB in this last its value ABC.

$$Ac = ABCc \cdot|\cdot Abc = Abc.$$

Similarly for B without C following

$$Bc = ABc \cdot|\cdot aBc = ABCc \cdot|\cdot aBc = aBc.$$

45. Suppose a class S to be divided (1) on one principle into A and B, and on another principle (2) into C and D, the divisions being exhaustive; suppose further that (3) all A is C, and (4) all B is D; can you conclude that all C is A, and all D is B? [E.]

The meaning of this problem appears to be that the class S will, as regards A and B, consist of S A *b* and S *a* B, and similarly as regards C and D; if so, there will under the first two conditions be only four possible combinations, namely—

SA*b*C*d*.	S*a*BC*d*.
SA*bc*D.	S*a*B*c*D.

But the further condition (3) negatives SA*bc*D, and (4) negatives S*a*BC*d*, so that, on inquiring for the description of C, we find it is (within the class S), A*b*C*d*; similarly D is *a*B*c*D. Both questions then may be answered in the affirmative, provided that we are not to look beyond the sphere of the class S.

46. What are the classes of objects regarded as possessing or not possessing the qualities A, B, C, D, which may exist consistently with the fundamental Laws of Thought, and the conditions that no class possesses both A and B, and that everything which does not possess B possesses C but not D? [L.]

The first condition that no class possesses both A and B will be sufficiently expressed in the premise A = A*b*, which prevents A and B from meeting. The second condition is obviously *b* = *b*C*d*. On going over the sixteen combinations in the fifth column of the Logical Alphabet (p. 181), it will be obvious that the first four, containing AB, are negatived by the first premise. The third four (*a*B) remain untouched: of the second and fourth fours containing *b*, all are negatived except A*b*C*d* and *ab*C*d*. The adjoining list of combinations is therefore the answer to the question.

A*b*C*d*
*a*BCD
*a*BC*d*
*a*B*c*D
*a*B*cd*
*ab*C*d*

47. How can we represent analytically the precise meaning of the opposition between a universal affirmative proposition and its contradictory, say between All As are Bs, and some As are not Bs?

The universal affirmative is symbolised as A = AB, and its logical power is to negative the combination A*b*, as shown in the margin. Now 'some As are Bs' was before explained to mean 'one A at least, it may be more or all As.' But, even if there be one A*b* found, it establishes the existence of the combination, subject to remarks elsewhere made (p. 142). In this qualitative treatment of logic number enters not at all, so that one counts for as much as a million. The force of the particular negative proposition is, then, to restore the combination which had been removed by the universal affirmative.

AB
$\boxed{Ab}$
*a*B
ab

48. If to the premises of an affirmative sorites we add a proposition affirming the first subject of

> the last predicate, the conditions now become equivalent to an equally numerous series of identities, or doubly universal propositions in Thomson's form **U**.

Symbolically, if we have the series of premises A = AB, B = BC, C = CD, and so on, up to X = XY, and we then add the condition Y = AY, the premises immediately become the same in logical force as

$$A = B = C = D = \ldots\ldots = X = Y.$$

To give a perfect demonstration of this theorem might not be very easy; but the student may convince himself of its truth by observing in several trials that the combinations consistent with the premises of a sorites as shown above, never contain a negative letter to the right hand of a positive one in the usual order of the alphabet. Thus the combinations consistent with the first two premises are

ABC, *a*BC, *ab*C and *abc*;

those for the first three are

ABCD, *a*BCD, *ab*CD, *abc*D and *abcd*.

Hence the last predicate appears in every combination except the last, and the first subject only in the first combination. In affirming the first subject of the last predicate, then, all combinations except the first, which contains both terms, and the last, which contains neither, must disappear. There remain in every case only the two combinations ABCDE . . . . XY . . . . and *abcde* . . . . *xy* . . . . which proceed from the identities stated in the question.

Suppose a pillar of circular section to be so shaped that no lower section is of less diameter than any upper section, but the section at the bottom is not greater than the section at the top ; we have here a physical analogue to the heap of propositions described above.

49. Is Professor Alexander Bain correct in the following extract from his *Deductive Logic* (p. 159)?

(1) 'Socrates was the master of Plato.
(2) Socrates fought at Delium.
(3) The master of Plato fought at Delium.

'It may fairly be doubted whether the transitions, in this instance, are anything more than equivalent forms. For the proposition (4) "Socrates was the master of Plato, and fought at Delium," compounded out of the two premises, is obviously nothing more than a grammatical abbreviation. No one can say that there is here any change of meaning, or anything beyond a verbal modification of the original form.'

Professor Bain in writing the above was clearly in need of means of more accurate analysis than his logical studies had afforded him. For if we put

A = Socrates; B = master of Plato;
C = one who fought at Delium,

the premises are certainly

$$(1) \quad A = B ;$$
$$(2) \quad A = AC.$$

The conclusion (3) as it stands is $B = BC$, which negatives only two combinations ABc and aBc, whereas the premises negative *in addition* the three A*b*C, A*bc*, *a*BC. It is possible, indeed, to draw the conclusion (5) B = AC, which is better than (3) by two combinations, namely, A*b*C and *a*BC. As to the *supposed* proposition (4), it cannot be made into any non-disjunctive proposition without a change of meaning; for, whether we make it into (6) A = BC, or A = ABC, it differs in force from the premises (1) and (2), *which propositions in fact cannot be condensed into any single non-disjunctive proposition of equivalent meaning.* The fact is that the supposed proposition (4) consists merely of the two (1) and (2) re-stated in one compound sentence. It is not 'the proposition' at all; it is 'the propositions.' The case would have been considerably altered, indeed, had Mr. Bain interpreted (1) as 'Socrates was *a* master of Plato,' of the form A = AB. The type of the premises would then have been essentially altered; but that he does not so interpret it is obvious from (3), in which we have 'the master,' not 'a master.' Altogether this page of Professor Bain's work affords remarkable evidence of the inability of a most acute logician to maintain accuracy of logical vision without the aid of some kind of calculus like that developed in the latter part of this work.

I append logical diagrams which almost explain themselves, the combinations pointed out by each bracket being those negatived by the proposition whose number is attached to the bracket.

	ABC			ABC	
	ABc			ABc	
(2)	AbC		(5)	AbC	(6)
	Abc	(3)		Abc	
	aBC			aBC	
(1)	aBc			aBc	
	abC			abC	
	abc			abc	

50. How far does the conclusion of an Aristotelian syllogism fall short of giving all the information contained in the premises?

The premises of Barbara, say A = AB, B = BC, negative four combinations, AB*c*, A*b*C, A*bc*, *a*B*c*. The conclusion A = AC negatives only two of these, namely, AB*c* and A*bc*. Measured in this way, then, it contains only half of the information of the premises; but of course if the conclusion gives just that information which is desired, the overlooking of the rest is no harm. Enough is as good as a feast—or rather better.

51. Take the premises of a syllogism in Barbara, such as (1) all As are Bs, and (2) all Bs are Cs, and determine precisely what they affirm, what they deny, and what they leave in doubt.

ABC
AB*c* (2)
(1) A*b*C
(1) A*bc*
*a*BC
*a*B*c* (2)
*ab*C
abc

To answer this question, we must form the eight combinations of A, B, C and their negatives, as in the margin; we then strike out A*b*C and A*bc* as being in conflict with condition (1), and AB*c* and *a*B*c* as being similarly in conflict with the condition (2), that all Bs are Cs. There remain four combinations, ABC, *a*BC, *ab*C, and *abc*. But these do not stand on the same logical footing, because if we were to remove ABC, there would be no such thing as A left; and if we were to remove *abc* there would be no such thing as *c* left. Now it is the Criterion or condition of logical consistency (p. 181) that every separate term and its negative shall remain. Hence there must exist some things which are described by ABC, and other things described by *abc*. But as regards the remaining two combinations, *a*BC and *ab*C, the case is different; for we may remove either, or both of these without wholly removing any term. We might add to the premises the new condition that all BCs are As, or BC = ABC, which would negative *a*BC; or we might add the condition, all Cs are As, or C = AC, which would remove both *a*BC and *ab*C.

We may sum up the meaning of the original premises (1) and (2) by saying that they deny the existence of AB*c*, A*b*C, A*bc*, and *a*B*c*; that they affirm the presence or *logical existence* of ABC and *abc*; and thirdly, while leaving *a*BC and *ab*C uncontradicted, they are consistent with the presence or absence of these two combinations. This is all that they leave in doubt concerning the relations of A, B, and C.

52. What is the amount of contradiction in the following celebrated epigram?

> 'The Germans in Greek,
> Are sadly to seek;
> *   *   *   *
> All, save only Hermann,
> And Hermann's a German.'

Putting A = German; B = Hermann;
C = sadly to seek in Greek,

the premises are evidently

(1) A = AC.
(2) B = B*c*.
(3) B = AB.

The logical diagram is as in the margin; it will be noticed that B disappears entirely, indicating contradiction; but A remains in the combination A*b*C. It is obvious that the wit of the epigram arises from the perception of contradiction. (See Hamilton's *Lectures*, vol. iii. p. 393.)

ABC (2)
AB*c* (1)
A*b*C (1)
A*bc* (1)
*a*BC (2) (3)
*a*B*c* (3)
*ab*C
abc

53. Show that you can make no assertion about two terms A and B (and these only), which is not either contained in the assertion of identity (A = B), or else contradictory thereto.

The proposition A = B removes two out of the four combinations thus—

Consistent Combinations.	Inconsistent Combinations.
AB.	A*b*.
ab.	*a*B.

Now, if any new assertion negatives either or both of A*b* and *a*B, it must be an assertion contained in and inferrible from A = B. If it removes either AB or *ab*, it must contradict A = B, because either A and B or *a* and *b* will then disappear entirely from the Logical Alphabet. It might be said perhaps that a new assertion could remove one consistent and one inconsistent combination, for instance, *ab* and A*b*; but this cannot be done except by a contradictory assertion. Any other pairs such as AB and A*b*, AB and *a*B, or *ab* and *a*B, being removed, removes some letter entirely and involves contradiction.

54. Is it (1) logically (2) physically possible that all material things are subject to the law of gravity, and that at the same time all not material things should be subject to the same law? [L.]

It is logically possible, that is to say, in accordance with the Laws of Thought, that all *things material* and all *things not material* should be subject to the law of gravity. In this case what is not subject to the law of gravity would be found among *not-things.* But it is not logically possible that all (material things) and all not-(material things) should be subject to the law of gravity, because this is equivalent to denying the existence of any class not subject to the law of gravity. This class would by one condition be not-material, and by the other condition it would be material, which is impossible. But by the law already described (p. 181) as the Law of Infinity, every logical term must be assumed to have its negative. The student is recommended to work out this question with the aid of letter symbols.

As to the second part of the question, what is not logically possible is of course not physically possible. Hence we are restricted to the inquiry whether it is physically possible that all things material and all things not-material should be subject to the law of gravity. This can only be answered on logical grounds thus far, that if the property of gravitation is essential to material things and forms a part of the definition of them, then it is not possible that not-material things should gravitate. As a matter of fact the possession of inertia is perhaps the ultimate test of materiality; but gravity is proportional to inertia and is an equally good test.

55. It is observed that the phenomena A, B, C occur only in the combinations AB*c*, *ab*C, and *abc*. What propositions will express the laws of relation between these phenomena?

Of the eight combinations of A, B, C, only these three remain. As we see that A occurs with and only with B, and *a* with and only with *b*, it is firstly obvious that A = B is the chief law. But as this law of relation leaves the combination ABC uncontradicted, we must have a second law to remove this, which may be either AB = AB*c*, or else B = B*c*. Observe, however, that the laws A = B and B = B*c* overlap and are pleonastic, because they both deny that B can be *a*BC. Hence the simplest statement of the laws of relation is

A = B.
AB = AB*c*.

56. Given three terms,—for instance, *water*, *blue*, and *fluid*,—how would you proceed to ascertain the utmost number of purely logical relations which can exist among them? [L.]

The relations of any three terms or things or classes of things must be governed in the first place by the universal Laws of Thought (p. 180). These laws restrict the combinations of three things, present or absent, to eight at the utmost; for each thing may be present or absent giving $2 \times 2 \times 2 = 8$ cases. But any special logical relation which may exist between the things has the effect of further restricting these combinations; the relation that water is a fluid, prevents the existence of the combination water, not-fluid. Conversely the removal from the series of any one or more of the eight combinations expresses the existence of a relation or relations negativing the existence of these combinations. Thus, the removal of the two combinations water, not-blue, fluid; water, not-blue, not-fluid, expresses the law that all water is blue. Thus the logical meaning of any condition is represented by the state of the combinations agreeing with those conditions. It follows that the utmost possible number of distinct logical relations will be ascertained by taking the eight possible combinations of the three terms and striking out one or more of the combinations in every possible variety of ways. The number of these ways cannot exceed 256; for each of the eight combinations may be either present or absent, giving $2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 = 256$ ways. But this calculation will include many cases where one or more of the three terms and their negatives disappear altogether, representing contradiction in the conditions. Many different selections, too, proceed from logical relations similar in character and form; thus the law $A = AB$ is similar to $A = Ab$, and to $a = ab$; the law $A = BC \cdot|\cdot bc$ is similar to $C = AB \cdot|\cdot ab$; and so forth. The investigation is fully described in the *Principles of Science* (pp. 134-143; 1st ed. vol. i. pp. 154-164) as also in the *Memoirs of the Manchester Literary and Philosophical Society*, Third Series, vol. v. pp. 119-130. It is found that the 256 possible selections are thus accounted for—

Proceeding from consistent logical conditions	192
" " inconsistent "	63
" " no condition at all	1
	256

The consistent logical conditions are found, however, on careful analysis to fall into no more than fifteen distinct forms, or *types of relation*, which are stated in the following table—

Reference Number.	Propositions expressing the general type of the logical conditions.	Number of distinct logical variations.	Number of combinations contradicted by each.
I.	A=B	6	4
II.	A=AB	12	2
III.	A=B, B=C	4	6
IV.	A=B, B=BC	24	5
V.	A=AB, B=BC	24	4
VI.	A=BC	24	4
VII.	A=ABC	24	3
VIII.	AB=ABC	8	1
IX.	A=AB, *a*B=*a*B*c*	24	3
X.	A=ABC, *ab*=*ab*C	8	4
XI.	AB=ABC, *ab*=*abc*	4	2
XII.	AB=AC	12	2
XIII.	A=BC ꖌ A*bc*	8	3
XIV.	A=BC ꖌ *bc*	2	4
XV.	A=ABC, *a*=*a*B*c* ꖌ *ab*C	8	5

CHAPTER XXII

ON THE RELATIONS OF PROPOSITIONS INVOLVING THREE OR MORE TERMS

1. THE doctrine of the opposition of propositions, exhibited in the well-known square, is an important and interesting fragment of ancient logic; but it is now apparent that propositions involving only two terms one subject and one predicate, do not sufficiently open up the question of the relationship of propositions. Two terms admit of only four combinations, and these can be present and absent only in sixteen ways, nine of which involve contradiction. There remain only seven cases of logical relation which resolve themselves into only two distinct types of proposition. (*Principles of Science*, pp. 134-7; 1st ed. vol. i. pp. 154-7.) With the introduction of a third term the sphere of inquiry becomes immensely extended. There are now, as we have seen (p. 221) 193 different cases of selection of combinations resolving themselves into fifteen distinct types of relation. The possible modes of relation of one proposition to another, including under the expression 'one proposition' any group of propositions, become considerably complex. Such modes of relation seem to be seven in number: thus one proposition is as regards another—

(1) Equivalent.
(2) Inferrible, or contained in the other, but not equivalent.
(3) Partially inferrible and otherwise consistent.
(4) Consistent but indifferent and not inferrible.
(5) Partially inferrible, partially contradictory.
(6) Partially indifferent, partially contradictory.
(7) Contradictory.

2. Let us take as an example the proposition

Steam = aqueous vapour,

and give a pretty complete analysis of its related propositions.

$$\begin{aligned} \text{Let}\quad \mathrm{A} &= \text{steam};\\ \mathrm{B} &= \text{aqueous};\\ \mathrm{C} &= \text{vapour}.\end{aligned}$$

The proposition being evidently of the form

$$(1)\quad \mathrm{A} = \mathrm{BC},$$

the combinations contradicted will be as in the margin.

$$\begin{array}{r} \mathrm{ABC} \\ (1)\left[\begin{array}{r} \mathrm{AB}c \\ \mathrm{A}b\mathrm{C} \\ \mathrm{A}bc \\ a\mathrm{BC} \end{array}\right. \\ a\mathrm{B}c \\ ab\mathrm{C} \\ abc \end{array}$$

The equivalent proposition will be—

Not steam = not aqueous or not vapour.

An inferrible but not equivalent assertion will be any one which negatives one, two, or three, but *not* four of the combinations negatived by (1). There will therefore be

$$4 + \frac{4 \times 3}{1 \times 2} + \frac{4 \times 3 \times 2}{1 \times 2 \times 3}$$ or 14 such inferrible and logically distinct propositions. We may infer—steam is aqueous; steam is vapour; what is not vapour is not steam; what is not aqueous is not steam; non-aqueous vapour is not steam; and so forth.

The third class of related propositions will include those which negative one or more of the excluded combinations, and one or more indifferent combinations. Indifferent combinations, as the name expresses, are those which can be removed without wholly removing any of the letters A, B, C, *a*, *b*, *c*. In this case any one of the remaining combinations except ABC may be singly removed. Thus 'not steam is not aqueous,' or in letters $a = ab$, is not contradictory to (1) and it may be inferred from (1) in respect of vapour which is not steam. But the assertion that other things which are not steam are not aqueous is not inferrible, but is consistent with (1). A proposition, again, which should negative A*b*C, A*bc*, *a*B*c*, *ab*C will be inferrible in respect of the two former, and consistent in respect of the two latter combinations. To ascertain what such proposition is we must look in the Logical Index, afterwards described, for the proposition which leaves $\alpha\ \beta\ \epsilon\ \theta$, and we find in the 55th place $b = ac$, or not-aqueous = not-steam and not-vapour. The other possible propositions of the same class are numerous and various.

To obtain one of the fourth class, which is merely consistent and indifferent, we must take any one or more of the combinations unnegatived by (1), for instance *ab*C, in such a way as not wholly to remove any letter. Thus $ab = abc$, or 'not-steam which is not aqueous is not vapour' is an assertion quite indifferent to (1). So is the assertion $a\text{B} = a\text{C}$ (Logical Index, No. 7).

Contradictory propositions being defined as those which wholly remove any term, such will be any one which removes ABC. Thus to say that steam is not aqueous is a case of the 5th class; it is inferrible from (1) in respect of steam which is not vapour (AB*c*), but it is contradictory because it also negatives steam which is vapour.

A proposition of the sixth class is discovered by taking any combination which may be spared with one which cannot, such as *ab*C and ABC, and looking in the index, we find AC = *b*C, or steam-vapour is identical with non-aqueous vapour, as a partially consistent, partially contradictory proposition as regards (1). It may or may not be true that what is not steam and not aqueous is not vapour, but it is contradictory to (1) to say that vaporous steam is not aqueous.

An example of a simply contradictory proposition of Class 7 is found in one which removes ABC only, such as AB = AB*c*; again *a* = *a*B, or not-steam is aqueous deletes *b*; *c* = AB*c* deletes *c*.

2. As a second example, let us take the propositions—(1) Hand = right-hand or left-hand; (2) Right is not left.

Putting A = hand; B = right; C = left; the conditions are evidently

$$(1)\ \mathrm{A} = \mathrm{AB} \cdot\!|\!\cdot \mathrm{AC}.$$
$$(2)\ \mathrm{B} = \mathrm{B}c.$$

The consistent combinations are shown in the margin, and the student may verify the following list, which gives one specimen of each of the seven classes of related assertions, the reference number of the Logical Index being also added.

ABC
AB*c*
A*b*C (2)
(1) A*bc*
*a*BC
*a*B*c*
*ab*C
abc

(1) Equivalent. B = B*c*; *bc* = *abc*. No. 153.
(2) Inferrible, etc. *a*B = *a*B*c*. No. 9.
(3) Partially inferrible, etc. *a* = *abc*. No. 15.
(4) Consistent, etc. AB = ABC; *ab* = *abc*. No. 67.
(5) Partially inferrible, etc. C = AC; A = AB. No. 59.
(6) Partially indifferent, etc. A = AB*c*; *ab* = *abc*. No. 179.
(7) Contradictory. *b* = *bc*. No. 35.

CHAPTER XXIII

EXERCISES IN EQUATIONAL LOGIC

I NOW give a small collection of examples and problems designed to enable the student to acquire a complete command of the equational and combinational views of logic. They are for the most part devised specially for this book, but a few have been utilised in examination papers, and a few have been adopted as indicated from the papers of other examiners. These questions form perhaps a partial answer to Professor Sylvester's remark, as quoted in the preface, especially when we observe that the questions and problems involving the relations of three terms can be multiplied almost *ad infinitum*, without resorting to like questions involving four, five, or more terms. The student will readily gather that the number, variety, and complexity of problems in pure logic is simply infinite, and is such as we gain no glimpse of in the old Aristotelian text-books.

1. Represent equationally the following assertions :—

(1) With the exception of porcelain there is no non-metallic substance which has been employed to make coins.

(2) With the exception of zinc and the metals discovered during the last hundred years, there is no metal which has not been employed to make coins.

(3) 'The worth of that is that which it contains,
And that is this, and this with thee remains.'
[SHAKESPEARE.]

(4) It is dangerous to let a man know how far he is but a brute, without showing him also his grandeur. It is dangerous again to let him see his grandeur, without his baseness. It is [even more] dangerous to leave him ignorant in both ways; but it is a high advantage to represent to him both the one and the other. (Pascal, *Pensées.*)

2. Represent in the forms of equational logic any of the following arguments:—

(1) Milton was a great poet, and a fearless opponent of injustice; a great poet should be honoured; a fearless opponent of injustice should be honoured: therefore Milton should be honoured.

(2) The virtues are either passions, faculties, or habits: they are not passions, for passions do not depend on previous determination; nor are they faculties, for we possess faculties by nature; therefore they are habits.

(3) There can be no person really fit to exercise absolute power, because the qualifications requisite to fit a person for such a position would consist in native talent combined with early training; now such a talent cannot be possessed in early childhood. (Suggested by De Morgan, *Syllabus*, p. 67.)

(4) One of the masters of chemistry was Berzelius;
Berzelius was a Swede;
One of the masters of chemistry was a Swede. [D.]

(5) This heavenly body is either a planet or a fixed star;

all fixed stars twinkle; planets do not twinkle; this body twinkles, therefore it is a fixed star.

(6) Show me any number of men, and I will say with confidence, either that they will with one accord raise their voices for liberty, or that there are aliens among them. (The stump orator's mode, according to De Morgan, of saying that all Englishmen are lovers of liberty.) [B.]

3. Infer all that you possibly can, by way of contraposition or otherwise, from the assertion 'all A that is neither B nor C is D.' [R.]

4. Express equationally Miscellaneous Example No. 39 in *Elementary Lessons in Logic*, p. 317.

5. What proposition concerning nebulae and vaporous bodies leaves doubtful the existence of a class of things which are neither nebulae nor vaporous bodies?

6. Represent the fact that A differs from B in two equivalent equational propositions.

7. Prove equationally that the proposition, All elements are either metal-elements or elements, is a mere truism.

8. What is the difference between the propositions $A = AB \cdot|\cdot A$, $B = AB \cdot|\cdot B$, and $A = B \cdot|\cdot A$?

9. Prove that if all not-Bs are not-As, and all Bs are As, then $A = B$, and *vice versâ*.

10. Show that the negative premises No As are Bs and no Cs are Bs, imply the logical existence of a class B which is neither A nor C.

11. Prove the equivalence of the following assertions:—

(1) Every gem is either rich or rare.
(2) Every gem which is not rich is rare.
(3) Every gem which is not rare is rich.
(4) Everything which is neither rich nor rare is not a gem.

12. Show that if metals which are either not valuable or not destructible are unfitted for use as money, it follows that destructible metals which are fitted for use as money must be valuable.

13. Does the proposition $A = B \cdot|\cdot BC$ differ in force from $A = B$?

14. 'All animals having red blood corpuscles are identical with those having a brain in connection with a spinal cord.' What is the description you may draw from this proposition of things having a brain not in connection with a spinal cord?

15. Luminous body is either self-luminous or luminous by reflection; melted gold is both self-luminous and luminous by reflection. Unmelted gold is not self-luminous but is luminous by reflection. Represent these premises symbolically, and draw descriptions of the terms (1) luminous body, (2) self-luminous body, (3) body luminous by reflection, (4) body not luminous, (5) body not self-luminous, (6) not melted gold, (7) not unmelted gold.

16. 'There are no organic beings which are devoid of carbon.' Determine precisely what this proposition affirms, what it denies, and what it leaves doubtful.

17. Prove the equivalence of the following statements—No right-angled triangles are equilateral; no equilateral triangles are right-angled; no right-angled equilateral figures are triangles.

18. All scalene triangles have their three angles equal to two right angles. What are the least or simplest assertions which added to the above will make it equivalent to 'All triangles are all figures which have their three angles equal to two right angles'?

19. All equal-sided squares have four right angles. What is the least extensive proposition which added to the above makes it equivalent to 'All squares are equal-sided and have four right angles'?

20. If an orator were to assert that Afghanistan is a very poor country, but it is essential to the security of India, but a reporter were to consolidate these two assertions into the one assertion that a very poor country, Afghanistan, is the Afghanistan which is essential to the security of India, how far would the reporter have misrepresented the logical meaning of the orator?

21. Express the following argument equationally:—Every organ of sense has nervous communication with the brain; for such is the case with all the five organs of sense, the eye, ear, nose, tongue, and skin.

22. If requested to draw from the assertion 'All coal contains carbon' a description of the term 'metal,' what answer would you give?

23. What values will you obtain for the terms man, brute, and gorilla, under the conditions that a gorilla is a man, and that all men are included and all gorillas excluded from the class of non-brutes?

24. Assuming that armed steam-vessels consist of the armed vessels of the Mediterranean and the steam-vessels not of the Mediterranean, inquire whether we can thence infer the following results:—

(1) There are no armed vessels except steam-vessels in the Mediterranean.
(2) All unarmed steam-vessels are in the Mediterranean.
(3) All steam-vessels not of the Mediterranean are armed.

(4) The vessels of the Mediterranean consist of all unarmed steam-vessels, any number of armed steam-vessels, and any number of unarmed vessels without steam. (Boole, 'The Calculus of Logic,' *Cambridge and Dublin Mathematical Journal*, 1848, vol. iii. pp. 199-201.)

25. How would you otherwise describe the class of things which are excluded from the class of white, malleable, metals?

26. Show that the description of the class of things which are not (either A, or if not A then both B and C), is as follows—either not-A and not-C, or if it be C then both not-A and not-B.

27. How do any two of the three equations $A = B$ $B = C$, $C = A$, differ in logical force from the third?

28. Frame a sorites with one premise negative and one particular, and represent it equationally.

29. Contrast the logical force of each of the propositions $A = AB \cdot|\cdot AC \cdot|\cdot AD \cdot|\cdot \ldots$ and $A = ABCD \ldots$, with that of the group of propositions $A = AB$, $A = AC$, $A = AD$, etc.; point out, moreover, which can be inferred from which.

30. Show that, under the condition of our Criterion of Logical Consistency (p. 181) the assertion that there are no such things as fresh-water foraminifera, involves the assertion that there are foraminifera which are not fresh-water beings, and fresh-water beings which are not foraminifera, but leaves doubtful the occurrence of things which are neither fresh-water beings nor foraminifera.

31. From the premises, 'All gasteropods are mollusca, and no mollusca are vertebrates,' obtain descriptions of the classes gasteropods and invertebrates.

32. 'Eloquence should contain both what is agreeable, and what is real; but what is agreeable should be real' (Pascal, *Pensées*). Represent the above symbolically, putting A = component of eloquent speech, B = agreeable, C = real.

33. Assuming it to be known that all mammals have red blood corpuscles, and that they also have vertebrae, invent five or six other distinct assertions which you might make about mammals, the possession of red blood corpuscles, and the possession of vertebrae, including of course the negatives of these terms, without coming into logical conflict with the known relations of the terms as above stated.

34. How would you otherwise describe the class of things which are excluded from the class of non-crystalline solids which are either non-metallic non-conductors, or else metallic conductors, and which are moreover either brittle and in that case useless for telegraphy, or else malleable and in that case useful for telegraphy?

35. Compare the following propositions :—

(1) X is Y.

(2) X is Y and is in some cases Z, and in some cases not Z.

By the law of excluded middle we know that X must be either Z or not Z. Is then the sentence (1) precisely identical in logical force with (2)? Compare now the following definitions :—

(3) A right-angled triangle is that which has a right angle.

(4) A right-angled triangle is that which has a right angle, and of which two sides are or are not equal.

Are these definitions precisely identical in logical force?

[C.]

36. What is the difference between saying that sea-water is drinkable and not scarce, and saying that drinkable sea-water is not scarce?

37. If from the premises 'All rectangles are parallelograms,' and 'Parallelograms consist of all four-sided figures whose opposite sides are parallel,' we infer that all rectangles are parallelograms, being four-sided figures with opposite sides parallel, how far does this inference fall short of being equivalent to the premises?

38. To say that Adam Smith is *the* father of Political Economy and a Scotchman is as much as to say that he is *a* Scotch father of Political Economy, and that no one but he can be *a* father of the science. Give the symbolic proof of this equivalence.

39. To lay down the condition that what is either A or else B, is what is both A and B or else both A and C and *vice versâ*, is to state disjunctively what may be laid down in two non-disjunctive propositions asserting that A without B is C and also B must be A.

40. Reduce the two assertions $A = Abc$ and $a = ac$ to a single one.

41. Give a good many inferences from the proposition $A = B \cdot|\cdot AC$, and also equivalents, distinguishing carefully between those inferences which are equivalent and those which are not.

42. Develop symbolically the term Plant (A) with reference to the undermentioned terms (B, C, D, E, F), under the conditions that acotyledonous (b) plants are flowerless; (c) monocotyledonous (D) plants are parallel-leaved (E); dicotyledonous (F) plants are not parallel-leaved; and

every plant is either acotyledonous, monocotyledonous, or dicotyledonous, but one only of these alternatives.

43. Completely classify triangles under the following conditions—

(1) Equilateral triangles are isosceles.
(2) Scalene triangles are not isosceles.
(3) Obtuse-angled triangles are not right-angled.
(4) Acute-angled triangles have three acute angles.
(5) Obtuse-angled triangles have not three acute angles.
(6) Equilateral triangles are not right-angled.

What other conditions must be added to comply with the results of geometrical science?

44. Among plane figures the circle is the only curve of equal curvature. Show that this is the same as to assert that a plane figure must either be a curve of equal curvature, in which case it is also a circle, or else, not a circle and then not a curve of equal curvature.

45. Which of the following propositions are equivalent to the first in the list?

(1) Crystallised carbon is not a conductor.
(2) Carbon which conducts is not crystallised.
(3) Conducting crystallised substance is not carbon.
(4) Conductors are either not carbon or not crystallised substances.
(5) Carbon is either not a conductor or not crystallised.
(6) Conductors which are not carbon are crystallised.
(7) Crystals are either non-conductors or not composed of carbon.
(8) Crystallised conductors are carbon.

46. Prove that any set of exclusive alternatives combined with part of that set produces only that part.

47. Show that the conclusions of Celarent, Cesare, Camestres, and Camenes give in each case only half the information contained in the premises.

48. Verify by various trials the statement that no inference by substitution within a group of propositions can negative combinations not negatived by the group of premises.

49. Show that Cesare and Camestres belong to the same type of assertion as Barbara and Celarent.

50. Assign the premises of the following moods of the Syllogism to their proper types of assertion:—Darapti, Bramantip, Camenes.

51. Prove that any proposition which is contradictory to 'common salt = sodium chloride,' can be inferred, so far as it is contradictory, from the assertion 'common salt = what is not sodium chloride.'

52. Does it or does it not follow that any proposition of the mth type (see pp. 221-2) will always be equally contradictory to one of the nth type?

53. Refer to Boole's *Laws of Thought*, pp. 146-9, and taking the premises of the complex problem there solved to be expressed in our system as follows:

$$(1)\ ac = ac\mathrm{E}\,(\mathrm{B}d \cdot|\cdot b\mathrm{D});$$
$$(2)\ \mathrm{AD}e = \mathrm{AD}e\,(\mathrm{BC} \cdot|\cdot bc);$$
$$(3)\ \mathrm{A}\,(\mathrm{B} \cdot|\cdot \mathrm{E}b) = \mathrm{C}d \cdot|\cdot c\mathrm{D};$$

work out the consistent combinations, and infer descriptions of the classes B, AC, ACe, D, e, AB, ABe, ab, AE, ACE, BD, DE, De, C, CD, etc. Verify by showing that D and e multiplied together give De and so forth.

54. If Brown asserts that all metals are reputed elements, and that all reputed elements will be ultimately decomposed, whereas Robinson holds that all metals are reputed elements

which will be ultimately decomposed, what is the exact amount of logical difference between them?

55. Compare the logical force of all the following propositions, and point out which pairs are equivalent, and which may be inferred from other ones.

(1) A square is an equal-sided rectangle.
(2) What is not equal-sided is not square.
(3) What is not square is not equal-sided.
(4) Equal-sided rectangles are squares.
(5) No rectangle which is not equal-sided is square.
(6) A square can be neither unequal-sided nor anything but a rectangle.
(7) An unequal-sided square does not exist.

56. Taking letters to represent qualities thus:—A = having metallic lustre; B = malleable; C = heavier than water; D = white coloured; E = fusible with difficulty; F = conducting electricity; form descriptions of each of the metals—gold, silver, platinum, copper, iron, lead, tin, zinc, antimony, sodium, and potassium, and then exhibit the extension of the following classes:—AB; BC; BCD; BCF; A*b*; *be*; B*d*; and so forth.

57. Express symbolically the following classes of things—

(1) Hard, wet, black, round, heavy, stone.
(2) Thing which is hard, wet, either black or red, but not round, and either heavy or not heavy.
(3) Thing which is either not hard, or not wet, or not a stone, but is either black and then round, or heavy and then a stone.

58. Referring to the *Principles of Science* (pp. 75-76; 1st ed. vol. i. p. 90), develop all the alternatives of A as limited by the description

$$A = AB\,\{C \cdot|\cdot D\,(E \cdot|\cdot F)\}$$

and infer descriptions of the following terms, Ace, Acf, $ABcD$. (See De Morgan, *Formal Logic*, p. 116; *Third Memoir on the Syllogism*, p. 12 in the *Camb. Phil. Trans.*, vol. x.)

59. Represent this argument symbolically:—A straight line can cut a circle in two points, and similarly an ellipse, and a hyperbola; but these are all the possible kinds of conic sections; therefore a straight line can cut any conic section in two points.

60. It being understood (1) that only the congenitally deaf are mute; (2) that an uneducated deaf person is mute, but uses signs; (3) that an educated deaf person is not mute, and does not use signs: express these conditions symbolically and describe the classes of persons who are deaf; mute; deaf-mutes; educated persons, etc.

61. Show how by the process of substitution alone to sum up into one disjunctive proposition the assertion that John is mortal; Thomas is mortal; William is mortal.

62. Prove that the *premises* of syllogisms in the moods Darapti and Felapton can be expressed in the form of a single non-disjunctive proposition, and assign its type. Show also that this is not the case with the moods of the other three figures.

63. Prove that the following propositions or groups of propositions involve self-contradiction:—

(1) $A = B \cdot|\cdot b$.

(2) $\left\{ \begin{array}{l} B = AB \\ b = Ab. \end{array} \right.$

(3) $A = AB$; $B = BC$; $C = aC$.

64. Analyse the force of Hamilton's form of proposition, 'Some A is not some A,' putting for 'some' and 'some' respectively the letter terms P and Q.

65. What does the assertion 'Some things are neither A nor B' tell us about things which are not-A?

66. How far do the conclusions of the syllogisms in Darapti, Felapton, Bramantip, Camenes, and Fesapo, as deduced on p. 188, respectively fall short of containing all the information given in the premises?

67. Show that $C = AbC \cdot|\cdot aBC$ is equivalent to the two propositions, $AB = ABc$ and $ab = abc$. Name the type.

68. To say that whatever is devoid of the properties of A must have those either of B or of D, or else be devoid of those of C, is the same as to say that what is devoid of the properties of B and D, but possesses those of C, must have A. Prove this.

69. What statement or statements must be added to the proposition, 'What is not a square is either not equal-sided or not a rectangle,' in order to make the assertions in the whole equivalent to the definition of a square that it is an equal-sided rectangle?

70. What is the difference between the assertion $A = ABC$ and the pair of assertions $b = ab$, and $c = bc$?

71. Prove that from one of the propositions, $A = ABC$, and $AB = ABC$, we can infer the other, but not *vice versâ*, and point out which is the one which can be so inferred.

72. Give three logical equivalents to the proposition, $ACD = AbCD$.

73. Demonstrate the equivalence of $A = AB \cdot|\cdot AC$ with $Ab = AbC$, and with $Ac = ABc$.

74. Show how by substitution alone to obtain $A = AB$ from $A = ABCD$; also obtain $A = AC$ and $A = AD$. (*Principles of Science*, p. 58; 1st ed. vol. i. p. 69.)

75. Verify the statement that any set of alternative terms combined with the same set, reproduces that set—that is to say, show that AA = A when for A we substitute any one of the following terms :—

$$ABc \cdot|\cdot AbC\ ;$$
$$ABC \cdot|\cdot aBC \cdot|\cdot abc\ ;$$
$$ABc \cdot|\cdot AbC \cdot|\cdot aBC.$$

76. Show by trial that if in any pair of logically equivalent assertions such as $A = Ab$ and $B = aB$, we substitute for A and B any logical expressions, such as CD for A, and CE for B, and their negatives in like manner for the negatives of A and B, we always obtain new equivalent assertions.

77. As a further example of equivalent assertions take the following pair of propositions :—

$$\begin{cases} AB = ABC, \\ Ac = Abc, \end{cases}$$

and substitute as follows :—

$$A = PQ, \qquad B = Qr, \qquad C = PR.$$

78. Express $a = ab$ and $Ab = AbC$ in the form of a single disjunctive proposition. To what type does it belong?

79. Express equationally De Morgan's forms of proposition (*Formal Logic*, p. 62).

(1) Everything is either A or B ;

(2) Some things are neither As nor Bs.

80. Verify the identical equations—

$$A \cdot|\cdot B = A \cdot|\cdot aB\ ;$$
$$AB = A\ (a \cdot|\cdot B)\ ;$$
$$A \cdot|\cdot BC = (A \cdot|\cdot B)\ (A \cdot|\cdot C).$$

81. Verify the following equivalences as transcribed from De Morgan's *Syllabus*, p. 42 :—

$$\begin{array}{ll}
\left\{\begin{array}{l} A = (B \cdot|\cdot C)\ D, \\ a\ = bc \cdot|\cdot d\ ; \end{array}\right. &
\left\{\begin{array}{l} A = B \cdot|\cdot C)\ (D \cdot|\cdot E), \\ a\ =\ bc \cdot|\cdot de\ ; \end{array}\right. \\
\left\{\begin{array}{l} A = BC \cdot|\cdot D, \\ a\ = (b \cdot|\cdot c)\ d\ ; \end{array}\right. &
\left\{\begin{array}{l} A\ = B \cdot|\cdot C\ (D \cdot|\cdot E), \\ a\ = b\ (c \cdot|\cdot de)\ ; \end{array}\right. \\
\left\{\begin{array}{l} A = (B \cdot|\cdot CD)\ (E \cdot|\cdot FG), \\ a\ = bc \cdot|\cdot bd \cdot|\cdot ef \cdot|\cdot eg\ ; \end{array}\right. &
\left\{\begin{array}{l} A = B \cdot|\cdot C \cdot|\cdot bD. \\ a\ = bcd. \end{array}\right.
\end{array}$$

82. State all the propositions involving only the terms named which can be inferred from the equation, Stone = rock ; and all the propositions which are *equivalent* to this one, Stone = stone-rock.

83. Show how by the mere process of substitution you can draw the proposition A = AD from the three propositions A = AB, B = BC, and C = CD.

84. What propositions added to A = AD are exactly equivalent in meaning to A = AB, B = BC, and C = CD jointly?

85. If both A and B have the property C, but A never occurs where D is, and B never occurs where D is absent, what is your description of the class of things which are devoid of the property C?

86. The proposition $A = A\ (B \cdot|\cdot C)$ being equivalent to $b = ab \cdot|\cdot AbC$, verify this truth by showing that it holds good when for A we substitute the term $Pq \cdot|\cdot pQ$, for B the term QRS, and for C the term qRs.

87. If a person were, correctly or incorrectly, to define Members of Parliament (including Lords and Commons) as either peers not chosen by election, or else not-peers chosen by election, that is as much as to assert both that all members are non-elected peers and elected non-peers, as well as that

all who are not members comprise the two classes of persons who are neither peers nor elected persons, and those who being peers have been elected but cannot sit.

88. It is not correct to say that because what is not A, but is B, is also C, therefore everything that is both B and C is A; but what further conditions may be laid down about the same things which will render these propositions convertible?

89. Into what other equivalent forms might we throw the joint statements that Venus is a minor planet, and minor planets are all large bodies revolving round the sun in slightly elliptic orbits within the earth's orbit?

90. If B is always found to coexist with A, except when X is Y (which it commonly, though not always, is), and if, even in the few cases where X is not Y, C is never found absent without B being absent also, can you make any other assertion about C? [R.]

91. If whenever X is present, Z is not absent, and sometimes when Y is absent, X is present, but if it cannot be said that the absence of X determines anything about either Y or Z, can anything be determined as between Z and Y? [R.]

92. If it is false that the attribute B is ever found coexisting with A, and not less false that the attribute C is sometimes found absent from A, can you assert anything about B in terms of C? [C.]

93. Referring to the *Elementary Lessons in Logic*, p. 196, from the premises there given ($A = B \cdot|\cdot AC$, $B = BD$, $C = CD$), derive descriptions of the terms BC, *a*, *b*, *d*.

94. From the important problem of Boole, described on p. 197 of the same lesson, with the premises $A = CD$, $BC = BD$, derive descriptions of the terms BC, *b*C, B, *b*, *d*.

95. In reference to this last named problem, examine each of the following assertions, and ascertain which of them are consistent with the premises A = CD, BC = BD

(1) $ac = ac\mathrm{D}$.	(4) $cd = acd$.
(2) $a = acd$.	(5) $\mathrm{A}b = \mathrm{A}b\mathrm{CD}$.
(3) ACD = ABCD.	(6) $abc = abcd$.

96. The premises AB = ABC, A = AB, and $\mathrm{A} = \mathrm{A}c$, involve self-contradiction. What is the least alteration which will remove this contradiction?

97. If AB = CD, what is the description of BD, of bd and of cd?

98. What must we add to the premises, All As are Bs and all Bs are Cs, in order that we may establish the relation that what is not A is not C?

99. Verify the assertion (*Principles of Science*, p. 141; first edit. vol. I. p. 162) that the six following propositions are all of exactly the same logical meaning:

$\mathrm{A} = \mathrm{BC} \cdot\!|\!\cdot bc$	$a = b\mathrm{C} \cdot\!|\!\cdot \mathrm{B}c$.
$\mathrm{B} = \mathrm{AC} \cdot\!|\!\cdot ac$	$b = \mathrm{A}c \cdot\!|\!\cdot a\mathrm{C}$.
$\mathrm{C} = \mathrm{AB} \cdot\!|\!\cdot ab$	$c = a\mathrm{B} \cdot\!|\!\cdot \mathrm{A}b$.

100. Write out five similar logical equivalents of the proposition $r = \mathrm{PQ} \cdot\!|\!\cdot pq$.

101. Prove that $ab = ab\mathrm{C}$ is equivalent to $ac = ac\mathrm{B}$, and $\mathrm{AB} = \mathrm{AC}$ to $\mathrm{A} = \mathrm{ABC} \cdot\!|\!\cdot \mathrm{A}bc$.

102. How may the condition $\mathrm{A} \cdot\!|\!\cdot \mathrm{B} = \mathrm{ACD} \cdot\!|\!\cdot \mathrm{BCD}$ be expressed in four non-disjunctive equations?

103. Verify the equivalence of $\mathrm{M} = \mathrm{M}n$ and $\mathrm{N} = \mathrm{N}m$

when for M and N we substitute successively the following pairs of values :—

$$(1) \begin{cases} M = A, \\ N = ABC. \end{cases} \qquad (2) \begin{cases} M = Ab, \\ N = cD. \end{cases}$$

$$(3) \begin{cases} M = ACD \cdot|\cdot AbcD \cdot|\cdot abCd, \\ N = bD \cdot|\cdot Bcd. \end{cases}$$

104. Express each of the following propositions equationally in a series of non-disjunctive propositions :

(1) Either the king is dead, or he is now on the march.

(2) Either compression or expansion will produce either heat or cold in a solid body.

(3) $Ab \cdot|\cdot bC = Cd \cdot|\cdot cD$.

(4) $AB \cdot|\cdot AC = (AB \cdot|\cdot AC)\ (Cd \cdot|\cdot cD)$.

105. In problem 20 (chap. xxi. p. 194) what description should we obtain of the classes *c*, those who do not take snuff, and *d*, those who do not use tobacco, respectively under the several conditions (1), (2) and (3), with (4)?

106. In problem 29, pp. 200–1, draw descriptions of the classes A*c*, *ab*, and *c*D.

107. Represent symbolically the logical import of the sentence: 'If it be erroneous to suppose that all certainty is mathematical, it is equally an error to imagine that all which is mathematical is certain.'

108. Represent equationally the logical import of this extract from the Oath of Supremacy: 'No foreign prince, prelate, person, state, or potentate, hath any jurisdiction, power, superiority, pre-eminence, or authority, ecclesiastical or spiritual, within this realm.' Observe especially how far the alternatives are or are not mutually exclusive.

109. Take the following syllogism in Datisi:

All men are some[i] mortals;
Some[ii] men are some[iii] fools;
Therefore, Some[iv] fools are some[v] mortals;

and analyse equationally the meanings of the word 'some' as it occurs five times. Show which of the 'somes' if any are exactly equivalent. Compare the result with the remarkably acute analysis of this mood given by Shedden, in his *Elements of Logic*, 1864, pp. 131–2.

110. If some Xs are Ys, and for every X there is something neither Y nor Z, prove that some things are neither Xs nor Zs. [DE MORGAN.]

111. Solve equationally Boole's example of analysis of Clarke's argument (see *Laws of Thought*, Chap. xiv.) The premises may be thus stated:

$$\begin{array}{ll} \left\{\begin{array}{l} \mathrm{ABD} = \mathrm{O}. \\ \mathrm{A}bd = \mathrm{O}. \end{array}\right. & \begin{array}{l} \mathrm{B}f = \mathrm{O}. \\ \mathrm{AF} = \mathrm{O}. \end{array} \\ \quad \mathrm{CDE} = \mathrm{O}. & \mathrm{A}e = \mathrm{O}. \end{array}$$

112. Show that every equational proposition whatsoever, the members of which are represented by X and Y in $X = Y$, may be decomposed into two propositions of the forms $X = XY$ and $Y = XY$, which will not however always differ. Show also that the operation when repeated gives no new result.

113. Take the definition Ice = Frozen Water, and throw it into equivalent propositions of the following forms:

(1) One disjunctive proposition.
(2) Two non-disjunctive propositions.
(3) One disjunctive and one non-disjunctive.
(4) Two disjunctives and one non-disjunctive.
(5) One disjunctive and two non-disjunctives.

(6) Three disjunctives.

(7) Four non-disjunctives.

Are these forms exhaustive, or can you frame yet other equivalent forms.

114. How many and what non-disjunctive propositions will be equivalent to the single disjunctive, $Ab \cdot|\cdot bC = Cd \cdot|\cdot cD$?

115. Express the proposition $AB = C \cdot|\cdot D$ in the form of two disjunctive and then in three non-disjunctive propositions.

116. As an exercise on Chapter XXII., take the proposition :

Stratified Rocks = Sedimentary Rocks, and discover (1) one equivalent; (2) two inferrible; (3) several partially inferrible and otherwise consistent; (4) several consistent, indifferent, and not inferrible; (5) two partially inferrible, partially contradictory; (6) one partially indifferent, partially contradictory; (7) one purely contradictory proposition.

117. Treat in the same general manner any of the following premises:

(1) Blood-vessels = arteries $\cdot|\cdot$ veins.

(2) Either thou or I or both must go with him.

(3) Heat is conveyed either by contact or radiation.

(4) An equation is either integrable or not integrable.

(5) Roger Bacon, an English monk, was the greatest of mediæval philosophers.

(6) Those animals which have a brain in connection with a spinal cord, and they alone, have red blood corpuscles. [MURPHY.]

118. Perform an exhaustive analysis of the relations of the following propositions, comparing each proposition

with each other in all the fifteen possible combinations, and ascertaining concerning each pair under which of the seven heads it falls:

(1) $A = BC$. (4) $a = BC \quad abc$.
(2) $Ab = Abc$. (5) $ab = ac$.
(3) $A = Ab$; $B = C$. (6) $AB = ABC$.

119. Perform a similar exhaustive analysis of the relations of the following propositions:

(1) Mercury = liquid, metal.
(2) Not-mercury is not liquid.
(3) Not-metal is either not-mercury or not-liquid.
(4) Mercury is a metal and is liquid.
(5) Liquid is either mercury or not-metal.
(6) Not-liquid is either not-mercury or metal.
(7) Not-mercury is either not-liquid or not-metal.

The eight propositions in question 45 or the seven in 55 of this chapter may be similarly analysed.

120. Analyse this argument: 'As we can only doubt through consciousness, to doubt of consciousness is to doubt of consciousness by consciousness.'

121. Illustrate the principle that the relations of logical symbols are independent of space-relations. (See *Principles of Science*, first ed. vol. i. pp. 39-42, 444; vol. ii. p. 469; new edition, pp. 32-35, 383, 769.)

122. Show that if certain premises involving three terms leave five or more combinations unnegatived, the premises in question must be self-consistent.

123. From the point of view of equational logic analyse

the metaphysical wisdom of Coleridge's doctrine of the syllogism thus expressed (*Table Talk*, vol. i. p. 207):

'All Syllogistic Logic is—1. *Se*clusion; 2. *In*clusion; 3. *Con*clusion;—which answer to the Understanding, the Experience, and the Reason. The first says: "This *ought* to be," the second adds: "This *is*," and the last pronounces: "This *must* be so."'

CHAPTER XXIV

THE MEASURE OF LOGICAL FORCE

1. THE combinational analysis of the meaning of assertions enables us to establish an almost mathematical system of measurement of the comparative force of assertions. Given the number of independent terms involved, that form of proposition has the least possible force which negatives only a single combination. Thus with three terms, a proposition of the form AB = ABC negatives only the single combination AB*c*; but A = ABC negatives three, and A = BC as many as four combinations. These latter propositions may be said to have three and four times the logical force of the first given.

2. I have not yet been able to discover any general laws regarding this subject of logical force, but many curious and perhaps important observations may be made. Thus a great many forms of assertion agree in having the logical force one-half, that is to say, they negative half the combinations. Such is the case, the terms being three in number, with the propositions A = BC; A = B ·|· BC; A = B*c* ·|· *b*C. Indeed, it is very frequently true that any proposition having no term common to both sides of the equation negatives half the combinations. This is true of all propositions of the types A = B, A = BC, and generally A = BCD . . . Y. But it is not true of the type AB = CD.

The appearance of the same term in both members of an equation always weakens its force; thus $A = ABC$ has the force only of $\frac{3}{8}$, whereas $A = BC$ has the force $\frac{1}{2}$. Again, $A = B \cdot|\cdot C$ has the force $\frac{1}{2}$, but $A \cdot|\cdot B = B \cdot|\cdot C$ only the force $\frac{2}{8}$.

3. The best ostensive instance of logical power is found in a form of proposition which embraces the greatest intension in one member with the greatest extension in the other. This kind of assertion has the general form $ABC \ldots = P \cdot\!:\!\cdot Q \cdot|\cdot R \cdot|\cdot \ldots$; and as the terms increase the logical force approaches indefinitely to unity. Thus while $A = B \cdot|\cdot C$ has the value $\frac{1}{2}$, $AB = C \cdot|\cdot D$ has that of 10 out of 16, and $A \cdot|\cdot B = CDE$ that of 22 out of 32.

A few other observations on this subject are thrown into the form of questions:

4. Show that the logical force of n equations of the form $A = B$, $B = C \ldots$ is $1 - \frac{1}{2^n}$.

5. Prove that a single proposition of the type $ABC \ldots = P \cdot|\cdot Q \cdot|\cdot R \cdot|\cdot \ldots$, there being in all n independent letter terms, and no term common to both members, has the logical force $1 - \frac{1}{2^{n-1}} + \frac{1}{2^{2n-1}}$ which approaches indefinitely to unity as n increases.

6. Can you discover any equation between a single term and any expression not involving that term which has a logical force other than one-half?

7. What form of proposition involving only A and B in one member, and C, D, in the other, has the lowest possible logical force?

8. What is the utmost number of combinations of n terms which can be negatived without producing contradiction?

9. What is the utmost number of combinations of four terms which can be negatived by a proposition involving only three of them?

10. What two propositions involving five terms negative the utmost possible number of combinations, without self-contradiction?

11. Show that m successive propositions of the type $A = AB$, $B = BC \ldots$, that is to say, in the form of the Sorites, leave $m + 2$ combinations unnegatived, so that the logical force is $1 - \frac{m+2}{2^{m+1}}$.

12. Prove that the amount of surplus assertion, or overlapping of the propositions, in a Sorites as treated in the last question, increases indefinitely. Investigate the law of the surplusage.

13. What is the utmost possible logical force as regards m terms of an equation involving n terms.

CHAPTER XXV

INDUCTIVE OR INVERSE LOGICAL PROBLEMS

1. THE direct or deductive process of logical analysis consists in determining the combinations which are, under the *Laws of Thought*, consistent with assumed conditions. The *Inverse Problem* is—given certain combinations inconsistent with conditions, to determine those conditions. As explained in the *Principles of Science* (chapter vii.) the inverse problem is always tentative, and consists in inventing laws, and trying whether their results agree with those before us. An American correspondent, Mr. M. H. Doolittle, points out that in making trials we should always pay attention to combinations in proportion to their *infrequency*, or solitariness, infrequency being the mark of deep correlation. The infrequency may be that either of presence or of absence.

2. The following inductive problems consist of series of combinations of three terms and their negatives which are supposed *to remain uncontradicted under the condition of a certain proposition or group of propositions.* The student is requested to discover such propositions, express them equationally, and then assign them to the proper type in the table on p. 222. If in any problem the conditions are self-contradictory the student is to detect the fact.

I.
ABC
abc

II.
A*bc*
*a*BC

III.
A*bc*
*a*BC
*a*B*c*

IV.
ABC
*a*B*c*
*ab*C

V.
A*b*C
A*bc*
*a*BC

VI.
A*b*C
A*bc*
abc

VII.
A*b*C
A*bc*
*a*BC
*a*B*c*

VIII.
ABC
*a*B*c*
*ab*C
abc

IX.
AB*c*
A*b*C
*a*BC
abc

X.
ABC
*a*BC
*a*B*c*
*ab*C

3. Assuming each of the following series of combinations to consist of those *excluded or contradicted by certain propositions*, assign the propositions which are just sufficient to exclude them in each problem, express these propositions equationally, and refer them as in the last question to the proper type in the table:

I.
ABC

II.
*ab*C

III.
*a*B*c*
*ab*C

IV.
ABC
abc

V.
ABC
A*b*C
A*bc*

VI.
AB*c*
A*bc*
abc

VII.
*a*B*c*
*ab*C
abc

VIII.
ABC
AB*c*
*a*BC
abc

IX.
AB*c*
A*b*C
A*bc*
*a*BC
*a*B*c*
*ab*C

4. I now give a series of inductive problems involving four terms. Each series of combinations consists of those which *remain* after the exclusion of such as contradict certain conditions. Required those conditions. The problems are ranged somewhat in order of difficulty.

I.

ABCD
abcd

II.

ABCD
ABC*d*
*a*BCD
*a*BC*d*
*ab*CD
*ab*C*d*
*abc*D
abcd

III.

ABC*d*
*a*BC*d*
*ab*C*d*
*abc*D
abcd

IV.

ABCD
AB*cd*
*ab*CD
abcd

V.

ABCD
ABC*d*
AB*cd*
A*b*CD
A*b*C*d*
abcd

VI.

ABC*d*
AB*cd*
A*b*C*d*
abcd

VII.

ABCD
ABC*d*
AB*c*D
A*bcd*
*a*B*cd*
abcd

VIII.

A*bca*
*a*BC*d*
*ab*C*d*
*abc*D
abcd

IX.

ABC*d*
*a*B*c*D
*a*B*ca*
*ab*C*d*

X.

AB*c*D
*a*BC*d*
*a*B*cd*
abca

XI.

AB*c*D
A*b*C*d*
A*bc*D
*a*BC*d*
*ab*C*d*
abcd

XII.

ABCD
AB*c*D
A*b*C*d*
*a*BC*d*
abcd

5. I next give a few similar problems involving five or six terms, as follows:

I.	III.	V.
ABCDE	ABC*d*E	AB*c*DE
AB*cde*	AB*cd*E	AB*c*D*e*
A*bcde*	A*b*C*d*E	AB*cd*E
*a*B*cde*	A*bc*D*e*	AB*cde*
abcde	*abcd*E	*ab*CD*e*
	abcde	*ab*C*de*

II.	IV.	VI.
ABC*de*	ABCDE	ABCDE
*a*BC*de*	A*b*C*de*	ABC*d*E
*ab*C*de*	*a*B*c*DE	AB*c*D*e*
*abc*DE	*ab*CD*e*	A*b*CD*e*
*abc*D*e*	*abcd*E	*a*B*c*DE
abcde		*ab*C*de*

VII.	VIII.
AB*c*D*ef*	ABC*def*
AB*cd*E*f*	A*b*CD*ef*
*a*B*c*D*ef*	*a*B*ca*EF
*a*B*cd*E*f*	*ab*CD*ej*
*ab*CD*e*F	*abc*DEF
*ab*CD*ef*	

IX.

AB*c*DEF	A*bc*D*ef*	*a*B*c*D*e*F	*ab*CD*ef*
AB*c*D*e*F	A*bcd*EF	*a*B*cd*EF	*abc*D*ef*
A*b*CD*ef*	A*bcde*F	*ab*CDEF	*abc*DE*f*
A*bc*DE*f*	*a*B*c*DEF	*ab*CD*e*F	*abcde*F

6. As the reader who is in possession of the present volume will have plenty of unanswered inductive problems, it may be well to give here the answers to the problems of the like kind which were set in the *Principles of Science*, new edition, p. 127. They are as follows:

I. $A = ABc \cdot\!|\!\cdot AbC$; $a = aBC$.

II. $A = AC$; $a = aB$.

III. $A = AC$; $ab = abc$.

IV. $A = D$; $B = CD \cdot\!|\!\cdot cd$, or their equivalents
$A = D$; $a = Bc \cdot\!|\!\cdot bC$.

V. $ab = abCd$; $Ab = AbD$; $ABc = ABcd$;
$aBC = aBCD$.

VI. $D = E$; $bC = bCD$; $(a \cdot\!|\!\cdot b)\ c = abcde$.

VII. $A = c = D = e$; $B = aB$.

VIII. (Unknown.)

IX. $bD = C \cdot\!|\!\cdot f$; $adE = BdF$; $ACF = ACdF$;
$bCDE = bCEF$.

X. This example was set by me at haphazard, like Nos. V. and VI., that is to say, by merely striking out any combinations of the logical alphabet which fancy dictated. Dr. John Hopkinson, F.R.S., has given me the following rather complex solution—

(1) $d = abd$.

(2) $b = b\ (AF \cdot\!|\!\cdot ae)$.

(3) $Af = AfBcDE$.

(4) $E = E\ (Bf \cdot\!|\!\cdot bACDF)$.

(5) $eB = eABCDF$.

(6) $abc = abcef$.

(7) $abef = abcef$.

Can a simpler answer be discovered?

7. In the first edition of the *Principles of Science*, vol. ii. p. 370, I gave a rather complex problem involving six terms, the combinations unexcluded being as follow—after coalescence of some alternatives:

ABCDF	AB*c*D*ef*
ABCD*ef*	*a*B*c*DF
ABC*d*E*f*	*a*B*c*D*ef*
AB*c*DF	*bcd*E*f*

To a request for solutions I received several answers, mostly correct, but, curiously enough, all differing in the forms of proposition. The correct ones are given below as furnishing a remarkable instance of logical equivalence.

First Answer.

$$\begin{aligned} C &= ABC \\ f &= Def \cdot|\cdot dEf \\ c &= BcD \cdot|\cdot bcd \\ d &= dEf \end{aligned}$$

Second Answer.

$$\begin{aligned} b &= cd \\ C &= ABC \\ D &= BD \\ d &= Ef \end{aligned}$$

Third Answer.

$$\begin{aligned} a &= ac \\ b &= ca \\ d &= Ef \end{aligned}$$

Fourth Answer.

$$\begin{aligned} C &= AC \\ b &= bcd \\ d &= Ef \\ B &= B\,(C \cdot|\cdot D) \end{aligned}$$

Fifth Answer.

$$\begin{aligned} AC &= BC \\ BDE &= BDEF \\ B &= C \cdot|\cdot D \\ d &= dEf \end{aligned}$$

The third answer, given by Mr. R. B. Hayward, M.A., is in the simplest terms. The propositions inserted as the fifth answer are those from which I formed the combinations deductively. The student may prove that any one of these answers is deducible from any other without descending explicitly to the combinations; thus $C = AC$ is the contrapositive of $a = ac$; $B = C \cdot\!|\!\cdot D$ is equivalent to $b = cd$, and so forth.

CHAPTER XXVI

ELEMENTS OF NUMERICAL LOGIC

1. LET a logical term, when enclosed in brackets, acquire a quantitative meaning, so as to denote the number of individual objects which possess the qualities connoted by the logical term. Then (A) = number of objects possessing qualities of A, or say, for the sake of brevity, the number of As.

Every logical equation now gives rise to a corresponding numerical equation. Sameness of qualities occasions sameness of numbers. Hence if $A = B$ denotes the identity of the qualities of A and B, we may conclude that $(A) = (B)$.

It is evident that exactly those objects, and those objects only, which are comprehended under A must be comprehended under B. It follows that wherever we can draw an equation of qualities, we can draw a similar equation of numbers. Thus, from $A = B = C$, we infer $A = C$; and similarly from $(A) = (B) = (C)$, meaning the number of As and Cs are equal to the number of Bs, we can infer $(A) = (C)$. But, curiously enough, this does not apply to negative propositions and inequalities. For if $A = B \sim D$ means that A is identical with B, which differs from D, it does not follow that

$$(A) = (B) \sim (D).$$

Two classes of objects may differ in qualities, and yet they may agree in number.

2. The sign ·|· being used to stand for the disjunctive conjunction *or*, but in an unexclusive sense, it follows that ·|· is not identical in meaning with +. It does not follow from the statement that A is either B or C, that the number o As is equal to the number of Bs added to the number of Cs; some objects, or possibly all, may have been counted twice in this addition. Thus, if we say *An elector is either an elector for a borough, or for a county, or for a university*, it does not follow that the total number of electors is equal to the number of borough, county, and university electors added together; for some men will be found in two or three of the classes.

This difficulty, however, is avoided with great ease; for we need only develop each alternative into all its possible subclasses and strike out any subclass which appears more than once, and then convert into numbers, connected by the sign of addition. Thus, from $A = B \cdot|\cdot C$ we get $A = BC \cdot|\cdot Bc \cdot|\cdot BC \cdot|\cdot bC$; but striking out one of the terms BC as being superfluous, we have $A = BC \cdot|\cdot Bc \cdot|\cdot bC$.

The alternatives are now strictly exclusive, or devoid of any common part, so that we may draw the numerical equation

$$(A) = (BC) + (Bc) + (bC).$$

Thus, if

A = elector,	C = county elector,
B = borough elector,	D = university elector,

we may from the proposition $A = B \cdot|\cdot C \cdot|\cdot D$ draw the numerical equation

$$(A) = (BCD) + (BCd) + (BcD) + (Bcd) + (bCD) + (bCd) + (bcD).$$

3. The data of any problem in Numerical Logic will be of two kinds:

(1) The logical conditions governing the combinations of certain qualities or classes of things, expressed in propositions.

(2) The numbers of individuals in certain logical classes existing under those conditions.

The *quaesita* of the problem will consist in determining the numbers of individuals in certain other logical classes existing under the same logical conditions, so far as such numbers are rendered determinable by the data. The usefulness of the method will, indeed, often consist in showing whether or not the magnitude of a class is determined or not, or in indicating what further hypotheses or data are required. It will appear, too, that where an exact result is not determinable we may yet assign limits within which an unknown quantity must lie.

4. In a certain statistical investigation, among 100 As there are found 45 Bs and 53 Cs; that is to say, in 45 out of 100 cases where A occurs B also occurs, and in 53 cases C occurs. Suppose it to be also known that wherever B is, C also necessarily exists. It is required to determine

(1) The number of cases (all being As) where C exists without B.

(2) The number of cases (all being As) where neither B nor C exists.

The data are as follow:

$$\text{Numerical equations}\begin{cases}(A) = 100 & \ldots\ldots\ (1)\\ (B) = 45 & \ldots\ldots\ (2)\\ (C) = 53 & \ldots\ldots\ (3)\end{cases}$$

Logical equation . . $B = BC$.

The logical equation asserts that the class B is identical with the class BC, which is the true mode of asserting that all Bs are Cs. Two distinct results follow from this, namely: 1st, that the number of the class BC is identical with the number of the class B; and 2nd, that there are no such things as Bs which are not Cs.

The logical equation is thus equivalent to two additional numerical equations, namely,

$$(B) = (BC) \quad \ldots\ldots\ldots \quad (4)$$
$$(Bc) = 0 \quad \ldots\ldots\ldots \quad (5)$$

We have now means of solving the problem; for, by the Law of Duality,

$$(C) = (BC) + (bC);$$

By (4)

$$= (B) + (bC).$$

Thus

$$53 = 45 + (bC),$$

or the required number of *b*Cs is 8.

To obtain the number of A*bc*s, we have

$$(A) = (ABC) + (ABc) + (AbC) + (Abc)$$
$$100 = 45 + 0 + 8 + (Abc).$$

Hence

$$(Abc) = 47.$$

5. The difference between the numbers of objects in any two classes whatsoever, is equal to the difference between the numbers of objects which are in each class, but excluded from the other class.

Take (A) and (B) to represent the numbers in any two classes A and B; then

$$\begin{aligned}(A) - (B) &= (AB) + (Ab) - (AB) - (aB)\\ &= (Ab) - (aB).\end{aligned}$$

6. If the number of As be x, of Bs be y, and of those Bs which are not As be p, then the number of As which are not B will be $p + x - y$.

Setting down the several logical quantities represented by $p + x - y$, we have

$$(aB) + (AB) + (Ab) - (AB) - (aB).$$

Four terms cross out, leaving only (Ab) as required.

7. Represent the following argument from Thomson's *Laws of Thought*, p. 168 :—

Three-fourths of the army were Prussians;
Three-fourths of the army were slaughtered;
Therefore some who were slaughtered were Prussians.

Taking A = members of the army,
B = Prussians,
C = slaughtered,

the premises are expressed as

$$(\mathrm{AB}) = \tfrac{3}{4}(\mathrm{A})$$
$$(\mathrm{AC}) = \tfrac{3}{4}(\mathrm{A}).$$

The number of Prussians slaughtered will be (ABC), of which the following equation is identically true :—

$$(\mathrm{ABC}) = (\mathrm{AB}) + (\mathrm{AC}) - (\mathrm{A}) + (\mathrm{A}bc)\,;$$

inserting values

$$\begin{aligned}(\mathrm{ABC}) &= \tfrac{3}{4}(\mathrm{A}) + \tfrac{3}{4}(\mathrm{A}) - (\mathrm{A}) + (\mathrm{A}bc)\\ &= \tfrac{1}{2}(\mathrm{A}) + (\mathrm{A}bc).\end{aligned}$$

That is to say, the number of Prussians slaughtered was at least half the army, and exceeds it by a number equal to the number of men in the army who were neither Prussians nor slaughtered.

8. If the number of As which are Bs is p, and the number of Bs which are Cs is q, what do we know concerning the number of As which are Cs?

We have the following self-evident equations :—

$$\begin{aligned}\mathrm{AC} &= \mathrm{ABC} + \mathrm{A}b\mathrm{C}\\ &= \mathrm{ABC} + \mathrm{AB}c + \mathrm{ABC} + a\mathrm{BC} + \mathrm{A}b\mathrm{C}\\ &\qquad + a\mathrm{B}c - \mathrm{B}\\ &= \mathrm{AB} \quad + \mathrm{BC} \quad - \mathrm{B} + \mathrm{A}b\mathrm{C} + a\mathrm{B}c.\end{aligned}$$

Inserting the values given, we get

$$\mathrm{AC} = p + q - \mathrm{B} + \mathrm{A}b\mathrm{C} + a\mathrm{B}c.$$

We see that the data are quite insufficient for determining the number of As which are Cs. They may be anything from zero up to the whole number of As or Cs. To make the question determinate we need also the number of Bs, as well as the number of ACs, which are not Bs, and the number of Bs which are neither A nor C.

9. Most Bs are As . . . . . . (1)
Most Bs are Cs . . . . . . (2)
Therefore, Some Cs are As . . . . . . (3)

The above argument is a celebrated one, proposed by De Morgan (*Formal Logic*, p. 163), and discussed by Boole (*Trans. of the Cambridge Philosophical Society*, vol. xi. part ii. p. 1) and others. Regarded as an ordinary Aristotelian pseudo-syllogism, it is subject to the fallacy of undistributed middle, since the proposition 'Most Bs are As,' must be counted as a particular affirmative. The pseudo-mood is accordingly **III** in the third figure. Nevertheless the force of the argument is pretty obvious and may thus be analysed.

The mark of quantity, *most*, of course means more than half, and is one of the few quantitative expressions used in ordinary language. We can easily represent the two premises in the form

$$(AB) = \tfrac{1}{2}(B) + w \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$
$$(BC) = \tfrac{1}{2}(B) + w' \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

To deduce the conclusion, we must add these equations together, thus,

$$(AB) + (BC) = (B) + w + w'.$$

Developing the logical terms on each side, we have

$$(ABC) + (ABc) + (ABC) + (aBC) = (ABC) + (ABc) + (aBC) + (aBc) + w + w'.$$

Subtracting the common terms, there remains

$$(ABC) = w + w' + (aBc).$$

The meaning is, that there must be some Cs which are As, amounting to at least the sum of the quantities w and w', the unknown excesses beyond half the Bs which are As and Cs. The number (aBc) is wholly undetermined by the premises, but it cannot be negative. Thus $w + w'$ is the lower limit of (ABC).

10. (1) For every Z there is an X which is Y; (2) Some Zs are not Ys. What inferences can be drawn?

This general problem given by De Morgan in his *Syllabus* (p. 29, art. 85), and in some other parts of his writings, would thus be represented in my formulae, which differ essentially, however, from those of De Morgan.

The premises are

$$(XY) = (Z) + m \quad . \quad . \quad . \quad . \quad . \quad (1)$$

in which m would be zero if De Morgan meant that there are not more Xs which are Ys than there are Zs, but just an equal number.

$$(Zy) = n \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

where n is some positive number.

Developing (1) we get

$$(3) \quad (XYZ) + (XYz) = (XYZ) + (XyZ) + (xYZ) + (xyZ) + m.$$

Striking out the common term and adding (Xyz) to both sides, we have for the number of Xs which are not Zs

$$(Xz) = m + n + (Xyz) + (xYZ).$$

Again, after striking out the common term, equation (3) reduces to

$$(XYz) = (XyZ) + (xZ) + m,$$

which gives as the number of Zs which are not Xs

$$(xZ) = (XYz) - (XyZ) - m.$$

The student should compare these results with those of the less general problem given in the *Principles of Science*, new edition, p. 169; first edition, vol. i. p. 191, and also with De Morgan's results expressed in a totally different kind of notation.

11. If m or more Xs are Ys, and n or more Ys are Zs, what do we know about the number of Xs which are therefore Zs?

This question represents one case of the numerically definite syllogism as treated by De Morgan (*Syllabus*, p. 27).

Taking X, Y, and Z to be the three terms of the syllogism, he adopts the following notation:—

u = whole number of individuals in the universe of the problem.
x = number of Xs.
y = number of Ys.
z = number of Zs.

Making m denote any positive number, mXY means, in De Morgan's system, that *m or more* Xs are Ys. Similarly nYZ means that *n or more* Ys are Zs. Smaller Roman letters denote the negatives of the larger ones. Thus mXy means that m or more Xs are not Ys, and so on.

From the two premises mXY and nYZ, De Morgan draws the conclusion $(m + n - y)$XZ. Let us consider what results are given by our notation. The premises may be represented by the equations

$$(XY) = m + m' \qquad\qquad (YX) = n + n',$$

where m and n are the same quantities as in De Morgan's system, and m' and n' two unknown but positive quantities, indicating that the number of XYs is m *or more*, and the number of YZs is n *or more*.

The possible combinations of the three terms X, Y, Z, and their negatives are eight in number, and these all together constitute the universe, of which the number is u. The problem is at once seen to be indeterminate in reality; for there are eight classes of which the numbers have to be determined, and there are only six known quantities, namely, u, x, y, z, m, and n, by which to determine them. Accordingly we find that De Morgan's conclusion, though not absolutely erroneous, has little or no meaning. From the premises he infers that $(m + n - y)$ *or more* Xs are Zs. Now

$$\begin{aligned} m + n - y &= (XY) + (YZ) - (Y) - m' - n' \\ &= (XYZ) - (xYz) \qquad - m' - n'. \end{aligned}$$

Thus De Morgan represents the number of the whole class, XZ, by a quantity indefinitely less than its own part, XYZ. It is quite true that if the second side $(XYZ) - (xYz) - m' - n'$ of this equation has value, there must be at least this number of Xs which are Zs; but as (xYz) may exceed (XYZ) in any degree, this may give zero or a negative result, while there is really a large number of XZs. The true and complete expression for the number of XZs is found as follows:—

$$\begin{aligned} (XZ) &= (XYZ) + (XyZ) \\ &= (XYZ) + (XYz) + (XYZ) + (xYZ) - (Y) + (XyZ) \\ &\quad + (xYz) = m + m' + n + n' - y + (XyZ) + (xYz). \end{aligned}$$

Among these seven quantities, only m, n, and y are definitely known. The two m' and n' are two indefinite quantities, expressing the uncertainty in the number of XYs and YZs, while there are two other unknown quantities, the numbers of XyZs and xYzs arising in the solution of the problem.

12. If m or more Xs are Ys, and n or more Ys are Zs, what do we know about the number of not-Xs which are not-Zs?

From the same two premises as in the last problem, namely

$$m\mathrm{XY} \text{ and } n\mathrm{YZ},$$

De Morgan draws the conclusion

$$(m + n + u - x - y - z)\mathrm{xz};$$

that is to say, the number of not-Xs which are not-Zs is the quantity in the brackets *or more.* This conclusion is equivalent to that in the preceding problem.

To prove this result it is requisite to develop all the combinations numbered in each of the quantities m, n, u, x, y, z; there are twenty-six terms in all which the reader may readily work out. Giving them the signs indicated by De Morgan, and striking out pairs of positive and negative terms, we find only two combinations left, together with m' and n', which terms are used, as in the last problem, to express the fact that De Morgan's proposition mXY is not really definite, but means that m or *more*, that is m or $(m + m')$ Xs are Ys. We thus obtain

$$(\mathrm{xy}) = (\mathrm{xyz}) - (\mathrm{XyZ}) - m' - n'$$

in which the term (XyZ) is wholly undetermined. Thus we find that De Morgan's method gives us as the value of (xy) a part of itself (xyz), diminished by three unknown quantities. The number (xz) may accordingly be of any magnitude, while the lower limit assigned to it by De Morgan is zero, or even negative. The problem is in fact a wholly indeterminate one, and De Morgan's solution is illusory.

Similar remarks may be made concerning other conclusions which De Morgan draws. Thus, from mXy and nYz (mXs or more are not Ys, and nYs or more are Zs) he infers

$$(m + n - x)\ \mathrm{xZ} \text{ and } (m + n - z)\ \mathrm{Xz}.$$

But it will be found by analysis that the first of these results has the following meaning :—

$$(\mathrm{xZ}) \overset{=}{>} (\mathrm{xYZ} - (\mathrm{XYz}) ;$$

that is to say, the lower limit of the class xZ is a part of itself, xYZ, diminished by the number of another class XYz of unknown magnitude.

13. If the fractions α and β of the Ys be severally As and Bs, and if $\alpha + \beta$ be greater than unity, it follows that some As are Bs.

[*Cambridge Phil. Trans.* vol. x. part i. p. 8.]

In his third memoir on the Syllogism De Morgan gives the above as a very general statement of the conditions of valid mediate inference. He remarks that the logician, that is to say, the ordinary Aristotelian logician, 'demands $\alpha = 1$ or $\beta = 1$, or both ; he can then infer.' This represents the condition of a distributed middle term.

The numerically definite conditions are readily represented as follow :—

The premises are $\qquad \alpha \,.\, (\mathrm{Y}) = (\mathrm{AY}).$

$\qquad\qquad\qquad\qquad \beta \,.\, (\mathrm{Y}) = (\mathrm{BY}).$

Hence

$$(\alpha + \beta)\ (Y) = (AY) + (BY)$$
$$= (ABY) + (AbY) + (ABY) + (aBY),$$
$$(\alpha + \beta)\ (Y) - (Y = (ABY) - (abY),$$

or

$$(ABY) = (\alpha + \beta - 1)\ (Y) + (abY).$$

We learn that the number of AYs which are Bs is the fraction $(\alpha + \beta - 1)$ of the Ys, together with the undetermined number (abY), which cannot be negative. But, according to the conditions, $\alpha + \beta$ is greater than unity; hence the second side of the equation must have a positive value. Not only will there be $(\alpha + \beta - 1)$, As which are Bs, but this is merely the lowest limit, and there will be as many more as there are units in the number of *ab*Ys.

If we distribute the middle term Y once, by making $\alpha = 1$, we have

$$(ABY) = \beta \,.\, (Y) + 0.$$

The term (abY) of course vanishes because the whole of the Ys are As. Again, if $\beta = 1$, we have

$$(ABY) = \alpha \,.\, (Y).$$

If both α and β become unity, then

$$(ABY) = (Y).$$

It must be carefully noted, however, that these results do not show the whole number of As which are Bs, but only those which are so within the sphere of the term Y. Nothing has been said about the combinations of not-Y, which are quite unlimited by the conditions of the problem.

14. 'If A occurs in a larger proportion of the cases where B is than of the cases where B is

not, then will B also occur in a larger proportion of the cases where A is than of the cases where A is not.'

This general proposition is asserted in J. S. Mill's chapter 'On Chance and its Elimination,' but is not proved by Mill. (*System of Logic*, Book III., chapter xvii. section 2, *ad finem;* fifth edition, vol. ii. p. 54.) I do not remember seeing any proof of it given elsewhere, and it is not to my mind self-evident. The following, however, is a proof of its truth, and is the shortest proof I have been able to find.

The condition of the problem may be expressed in the inequality

$$(AB) : (B) > (Ab) : (b),$$

or reciprocally in the inequality

$$(B) : (AB) < (b) : (Ab).$$

Subtracting unity from each side, and simplifying, we have

$$(aB) : (AB) < (ab) : (Ab).$$

Multiplying each side of this inequality by $(Ab) : (aB)$ we obtain

$$(Ab) : (AB) < (ab) : (aB).$$

Restoring unity to each side, and simplifying

$$(A) : (AB) < (a) : (aB),$$

or reciprocally

$$(AB) : (A) > (aB) : (a),$$

which expresses the result to be proved, namely, that B occurs in a larger proportion of the cases where A is than of the cases where A is not.

15. In a company of r individuals, p have coats and q have waistcoats. Determine some other relations between them.

Boole treats this problem in the fourth page of his Memoir *On Propositions Numerically Definite* (*Cambridge Philosophical Transactions*, vol. xi. part ii.). Taking 1 to represent the company which is the universe of the proposition, x the class possessing coats, y the class possessing waistcoats, and using the letter N, according to Boole's notation, as equivalent to the words 'number of,'

$$p = Nx, \quad q = Ny, \quad r = N1,$$

he finds, as we have found in a preceding page (p. 264, No. 7),

$$Nxy = p + q - r + N\overline{1-x}\ \overline{1-y}.$$

$$N\overline{1-x}\ \overline{1-y} = r - p - q + Nxy.$$

He proceeds, 'Again, let us form the equation

$$\begin{aligned} 2p - q - r &= 2Nx - Ny - N1 \\ &= N(2x - y - 1) \\ &= N(x\overline{1-y} - 2y\overline{1-x} - \overline{1-x}\ \overline{1-y}) \\ &= Nx\overline{1-y} - 2Ny\overline{1-x} - N\overline{1-x}\ \overline{1-y}. \end{aligned}$$

From which we have

$$Nx\overline{1-y} = 2p - q - r + 2Ny\overline{1-x} + N\overline{1-x}\ \overline{1-y}.$$

Hence we might deduce that the number who had coats but not waistcoats would exceed the number $2p - q - r$ by twice the number who had waistcoats without coats together with the number who had neither coats nor waistcoats. This is not, indeed, the simplest result with reference to the class in question, but it is a correct one.'

The student is requested to verify this result.

On going over this paper of Boole's again, it becomes apparent to my mind that his method is identical with that developed in this chapter and in my previous paper on the same subject (*Memoirs of the Manchester Literary and Philosophical Society*, Third Series, vol. iv. p. 330, Session 1869–70), written with a knowledge, as stated on p. 331, of Boole's publication on the subject.

16. Can we represent a syllogism in the extensive form by means of numerical symbols?

In a very interesting and remarkable paper read to the Belfast Philosophical Society in 1875, Mr. Joseph John Murphy has given a kind of numerical notation for the syllogism. He has since printed a more condensed and matured account of his views in *Mind*, January, 1877.

Taking the syllogism—'Chlorine is one of the class of imperfect gases; imperfect gases are part of the class of substances freely soluble in water; therefore, chlorine is one of the class of substances freely soluble in water'—he assumes the symbols

x = Chlorine, y = Imperfect gases,
z = substances freely soluble in water.

He expresses the first premise in the form

$$y = x + p,$$

p being a positive numerical quantity indicating that there are other things besides chlorine in the class of imperfect gases. The second premise takes the form

$$z = y + q,$$

similarly indicating that besides imperfect gases there are q things in the class of substances freely soluble in water. Substitution gives $z = x + p + q$,
which would seem to prove that besides chlorine (x) there are $p + q$ things in the class of substances freely soluble in water.

The student who wishes to master the difficulties of the modern logical views should read these papers with great care. Space does not admit of my arguing the matter out at full length, and I can therefore only briefly express my objections to Mr. Murphy's views as follows:—His equations are equations in extension, and, with his use of + and −, they can only hold true when his terms are numerical quantities. Under this assumption his equations show with perfect correctness the numbers of certain classes; but they are not therefore equivalent to syllogisms. Because $z = x + p + q$, we learn that the number z exceeds x by $p + q$, but it does not therefore follow that chlorine belongs to the class of substances represented by z. In short, as I have pointed out at the beginning of this chapter (p. 259), from logical equations arithmetical ones follow, but not *vice versâ*. (See also *Principles of Science*, p. 171; first edit. vol. i. p. 193.) I hold, therefore, that Mr. Murphy's forms are not really representations of syllogisms; but at the same time I am quite willing to admit that this is a question never yet settled and demanding further investigation. It is very remarkable that Hallam inserted in his *History of Literature* (ed. 1839, vol. iii. pp. 287–8) a long note containing a theory of the syllogism somewhat similar to that of Mr. Murphy, but which has hitherto remained unknown to Mr. Murphy and apparently to all other logical writers.

CHAPTER XXVII

PROBLEMS IN NUMERICAL LOGIC

1. If from the number of members of Parliament we subtract the number of them who are not military men, we get the same result as if from the whole number of military men we subtract the number of them who are not members of Parliament. Prove this.

2. In a company of x individuals it is discovered that y are Cambridge men, and z are lawyers. Find an expression for the number of Cambridge men in the company who are lawyers, and assign its greatest and least possible values.

[BOOLE.]

3. Prove that in any population the difference between the number of females and the number of minors is equal to the difference between the number of females who are not minors, and of minors who are not females.

4. Show that if to the number of metals which are red, we add the number which are brittle, the sum is equal to that of the whole number of metals after addition of the number of metals which are both red and brittle, and after subtraction of the number of metals which are neither red nor brittle.

5. What is the value of the following expression

$$(A) - (AB) - (AC)?$$

6. Prove that the number of quadrupeds in the world added to the number of beings not quadrupeds which possess stomachs is equal to the whole number of things having stomachs together with the number of things not having stomachs which are quadrupeds.

7. If x and y be respectively the numbers of things which are X and Y, while m is the whole number which are both X and Y, and n the number which are either X alone or Y alone, what is the relation between $m + n$ and $x + y$?

8. Let u be the whole number of things under consideration, x the number which are A, and y the number which are B; then if m be the number of things which are both A and B, show that $m + u - x - y$ is the number which are neither A nor B.

9. Taking each logical term to represent the number of things included in its class, verify the following equations:

$$(A - AB)(A - AC) = A - AB - AC + ABC = Abc.$$

$$(A - AB)(A - AC)(A - AD) = A - AB - AC + ABC - AD + ABD + ACD - ABCD = Abcd.$$

10. What is the product of the logical multiplication of the four factors

$$(A - AB)(A - AC)(A - AD)(A - AE)?$$

Give another expression for its value.

11. Show that the following equation is necessarily true:

$$B + AC + bC + Abc = A + C + aBc.$$

12. What happens in Problem 8 if it be discovered that the class B does not exist at all?

13. Find an expression for the difference between (A) and (B) + (C).

14. What is (α) the minimum percentage of C that *must*, and (β) the maximum that *may* coincide with B under the following conditions?

80 per cent of As coincide with 50 per cent of Bs.
70 per cent of As coincide with 60 per cent of Cs. [D.]

15. If revolutions occur in a larger proportion of governments where the press is under a censorship, than of governments where it is not, then will a censorship of the press be found in a larger proportion of governments which are subject to frequent revolutions, than of governments which are not thus subject. [D.]

16. If p per cent of A are B, and q per cent of A are C, what is the least percentage of A that those individuals make up which are both B and C? [D.]

17. Show that we cannot tell what percentage of B or of C the same individuals make up unless we know how much of B or of C is *not* A. [D.]

18. In the easy case in which *all* B is A, and all C is A, find what percentage of B or of C must be made up by the individuals which are both B and C at once. [D.]

19. Prove the following equations:

$$(\mathrm{A}bc) + (\mathrm{AB}) = (\mathrm{A}) + (\mathrm{ABC}) - (\mathrm{AC}).$$

$$(\mathrm{A} + \mathrm{B}) - (\mathrm{C} + \mathrm{D}) = (\mathrm{A} + \mathrm{B})\ (c + \mathrm{C}d) - (\mathrm{C} + \mathrm{D})\ (a + \mathrm{A}b) - \mathrm{AB}\ (\mathrm{C}d + c\mathrm{D}) + \mathrm{CD}\ (\mathrm{A}b + a\mathrm{B}).$$

20. Prove that the following equation gives a correct expression for the common part of any three classes – A, B, C.

$$(\mathrm{ABC}) = (\mathrm{B}) + (\mathrm{C}) - (\mathrm{A}) - (a\mathrm{B}) - (a\mathrm{C}) + (\mathrm{A}bc).$$

21. In a company consisting of r individuals there were q in number who knew Latin, and p in number who knew

either Latin or French, but not both; between what limits is the number of those who knew French confined?

22. In the last problem prove that the lower limit is the greatest value in $p-q$ and $q-p$, and the upper limit, the least value in $2r-p-q$, and $p+q$. (See Boole, *On Propositions Numerically Definite*, p. 15.)

23. The student will find many other numerically definite problems in De Morgan's *Formal Logic*, Chapter VIII., and in his *Syllabus*, pp. 27–30; but in reading De Morgan it must be carefully remembered that mXY means with him not that mXs are Ys but that *m or more* Xs are Ys. His solutions will sometimes, as shown in the previous chapter, be found delusive.

24. Verify the following assertion of De Morgan: 'To say that mXs are not any one to be found among any lot of nYs is a *spurious* (that is a self-evident or necessary) proposition, unless $m+n$ be greater than both x and y, in which case it is merely equivalent to both of the following, $(m+n-y)$ Xy, and $(m+n-x)$ Yx, which are equivalent to each other.

25. It is found that there are in a certain club of x members, y London graduates, and z lawyers. What further numerical data are requisite in order to define the numbers who are both London graduates and lawyers, and of those who are neither?

26. If there are more persons in a town than there are hairs on any one person's head, then there must be at least two persons in the town with the same number of hairs on their heads. Put this theorem into a strict logico-mathematical form. [HERBERT SPENCER.]

27. Demonstrate the theorem in numerical logic given in the *Principles of Science*, new edition (only), p. 170.

28. 'For every man in the house there is a person who

is aged; some of the men are not aged; it follows, and easily, that some persons in the house are not men; but not by any *common* form of syllogism.' (DE MORGAN, *Syllabus*, p. 29.) A solution of this problem is given in *Principles of Science*, new edition, p. 169.

29. Draw what conclusions you can from the following:

'There were some English on board; and though no passengers were saved from the wreck, and of the ship-officers, as it happened, only one, yet no Englishman was lost.' [R.]

CHAPTER XXVIII

THE LOGICAL INDEX

1. I NOW give what I propose to call the Logical Index, or, more precisely, the Logical Index of Three Terms. As however the logical relations of two terms are too simple to need an index, and those of four terms are vastly too numerous and complex to admit of exhaustive treatment at present, the Index of Three Terms is practically the only one which can be given. It contains, within the space of four pages, a complete enumeration of all possible purely logical conditions involving only three distinct terms.

2. Each page contains a double-sided table, forming in fact two tables. Each such table contains a column of equational propositions, a column of Roman numerals showing the *type* (see p. 222) to which such propositions belong, a column of consecutive Arabic numbers for sake of easy reference, and lastly a column of Greek letters, which supplement the Greek letters α, β, γ, given at the heads of the columns of propositions. These Greek letters stand in place of the combinations of the fourth column of the Logical Alphabet (p. 181), as follow:

$\alpha = \mathrm{ABC}$	$\epsilon = a\mathrm{BC}$
$\beta = \mathrm{AB}c$	$\zeta = a\mathrm{B}c$
$\gamma = \mathrm{A}b\mathrm{C}$	$\eta = ab\mathrm{C}$
$\delta = \mathrm{A}bc$	$\theta = abc$

It is obvious that each Greek letter appearing in the middle column of the Index represents the presence of the corresponding combination, or rather its *non-exclusion.* Absence of the Greek letter represents exclusion. Looking, for instance, to No. 31, we learn that $a = bc$, an assertion of the VIth type, excludes all combinations except α, β, γ, specified at the top of the table, and θ, specified in the centre column; that is to say, the combinations consistent with $a = bc$ are ABC, ABc, AbC, and abc. The principal use of the Index, however, will be in the inverse direction, to find the law corresponding to certain unexcluded combinations. Taking, for instance, the combinations Abc, aBC, abC, their Greek signs are δ, ϵ, η; to find their law, then, we must look in the last table in the column headed (not-α), (not-β), (not-γ), and in the line showing δ, ϵ, η, in the middle column. We there find the two assertions $\mathrm{A} = c = \mathrm{A}b$ of the IVth type (No. 230), as those corresponding to the combinations in question.

3. With the aid of this Index we can infallibly and rapidly solve all possible problems relating to three terms. What assertion, for example, can we make which shall not be contradictory to, and yet shall not be inferrible from, the premise $a = \mathrm{BC} \cdot\!|\!\cdot abc$? Working out the combinations unexcluded by this premise, we find them to be ABc, AbC, Abc, aBC, and abc, or β, γ, δ, ϵ, θ. Of these β and δ may be removed simultaneously without wholly removing any letter, that is to say without contradiction. Looking in the second table of the Logical Index at No. 81, we find the proposition $\mathrm{A} = \mathrm{AC}$ (of type II.) as one which removes β and δ. This is the required proposition which is, as it were, quite neutral to the one assumed. In the same way we might remove γ and θ without contradiction, so that No. 34, or $\mathrm{A}b = bc$, of type XII., is another neutral proposition. It

may I believe, be safely inferred that every proposition of type XIII., that of the premise in question, will have at least two propositions neutral to it, of types II. and XII. respectively.

Suppose it be required, as a second instance, to define the precise points of agreement and difference of two disputants, one of whom asserts that (1) 'Space = three-way spread with points as elements' (Henrici); while his opponent holds that (2) 'Space = three-way spread,' and at the same time (3) 'Space has points as elements,' but is not known to be the only thing that has. The three assertions are symbolised as below, the combinations excluded being indicated by their Greek signs:

$$(1)\ A = BC \left\{\begin{matrix}\beta\\ \gamma\\ \delta\\ \epsilon\end{matrix}\right. \qquad \begin{matrix}(2)\ A = B\\ (3)\ A = AC\end{matrix} \left\{\begin{matrix}\beta\\ \gamma\\ \delta\\ \epsilon\\ \zeta\end{matrix}\right.$$

We see that the second disputant's assertions have a logical force superior to that of the first by $\frac{1}{8}$, namely ζ, which corresponds to assertion 5, or $a\mathrm{B} = a\mathrm{BC}$. In addition, then, to all that the first asserts, he affirms that 'a three-way spread which is not space has points as elements.'

As a third instance of the power and flexibility of this combinational logic, suppose it to be required to make an exhaustive statement of all the inferences which can be drawn from the theorem that 'Similar figures (A) consist of all whose corresponding angles are equal (B), and whose corresponding sides are proportional (C).' We proceed in this way. The proposition is of the form A = BC, of the VIth type, and negatives β, γ, δ, ϵ. Any proposition, then, which negatives one, two, or three of these combinations will be inferrible from the theorem, but not equivalent to it. All the possible inferences, therefore, are indicated in the following table of Index Numbers, which, taken in connection with the Logical Index, sufficiently explains itself:

β......65	$\beta\gamma$......97	$\beta\gamma\delta$......113
γ......33	$\beta\delta$......81	$\beta\gamma\epsilon$......105
δ......17	$\beta\epsilon$......73	$\beta\delta\epsilon$...... 89
ϵ...... 9	$\gamma\delta$......49	$\gamma\delta\epsilon$...... 57
	$\gamma\epsilon$......41	
	$\delta\epsilon$......25	

These fourteen assertions, which are all the possible non-equivalent inferences, or the equivalents of these, were detected by the Logical Index in a few minutes; it would be doubtfully possible, and in any case a most laborious problem, to obtain an exhaustive statement of inferences by any other method, if indeed any other method exists.

The want of space alone prevents my giving more abundant illustrations of the multitudes of logical problems which may be solved infallibly and speedily by the use of the Logical Index. It may be safely said that in four pages of tables it gives the key to all possible logical questions, relations or problems involving three distinct logical terms. There is some possibility that the corresponding index for the relations of four terms may some day be worked out, as, when exhibited in like manner, it will occupy only one volume of 1024 pages of a rather larger size than those of this volume. There is no prospect whatever that the corresponding index for five terms will ever be exhaustively published, since it would fill a library of 65,536 volumes, each containing 1024 large pages. This fact will give some faint idea of the possible number and complexity of logical relations involving only a very moderate number of terms.

The Logical Stamp

In my previous logical books[1] I described a Logical Slate with five series of the combinations of the Logical Alphabet engraved upon it. I first made such a slate in May 1863, and I have since frequently used it with much saving of labour. The recent extensive introduction of india-rubber printing stamps lately suggested to me the idea that the most convenient method of obtaining the logical combinations would be to stamp them on paper. Two stamps producing the combinations of three and of four terms as shown in columns IV. and V. of the Logical Alphabet (p. 181), were made for me at a cost of about eleven shillings.

They have been very successful, and leave nothing to be desired as regards the private study of logical problems. One great advantage of the stamps over the slate is evident, namely, that the work being done on paper can be preserved for reference without copying.

The ABCD stamp can readily be utilised for problems of five, six, or more terms. For six terms, for instance, it is requisite to make four impressions and distinguish them by writing EF, E*f*, *e*F, *ef*, above the respective impressions.

India-rubber stamps of any design can now be easily ordered at all the principal stationers.

[1] *Pure Logic*, 1864, p. 68; *Substitution of Similars*, 1869, p. 54; *Elementary Lessons in Logic*, 1870, p. 199; *Principles of Science*, 1874, Vol. i. p. 110; *New Editions*, p. 96.

THE LOGICAL INDEX

Number.	Type.	Present α, β, γ.	Present.	Present α, β (not γ).	Type.	Number.
1		All present	$\delta\ \epsilon\ \zeta\ \eta\ \theta$	$Ab = Abc$	VIII.	33
2	VIII.	$ab = abC$	$\delta\ \epsilon\ \zeta\ \eta$	$Ab = bc$	XII.	34
3	VIII.	$ab = abc$	$\delta\ \epsilon\ \zeta\ \theta$	$b = bc$	II.	35
4	II.	$a = aB$	$\delta\ \epsilon\ \zeta$	$b = Abc$	VII.	36
5	VIII.	$aB = aBC$	$\delta\ \epsilon\ \eta\ \theta$	$Ab = Abc,\ aB = aBC$	XI.	37
6	II.	$a = aC$	$\delta\ \epsilon\ \eta$	$c = Ac,\ AC = ABC$	IX.	38
7	XII.	$aB = aC$	$\delta\ \epsilon\ \theta$	$b = bc,\ Bc = ABc$	IX.	39
8	VII.	$a = aBC$	$\delta\ \epsilon$	$b = bc,\ c = Ac$	V.	40
9	VIII.	$aB = aBc$	$\delta\ \zeta\ \eta\ \theta$	$AC = BC$	XII.	41
10	XII.	$aB = ac$	$\delta\ \zeta\ \eta$	$a = bC \cdot\!\!\mid\!\!\cdot aBc$	XIII.	42
11	II.	$a = ac$	$\delta\ \zeta\ \theta$	$C = ABC$	VII.	43
12	VII.	$a = aBc$	$\delta\ \zeta$	$a = aBc,\ Ab = Abc$	X.	44
13	II.	$a = ab$	$\delta\ \eta\ \theta$	$a = ab,\ Ab = Abc$	IX.	45
14	VII.	$a = abC$	$\delta\ \eta$	$a = bC$	VI.	46
15	VII.	$a = abc$	$\delta\ \theta$	$a = ab,\ b = bc$	V.	47
16		$a = 0$	δ	$a = 0$		48
17	VIII.	$Ab = AbC$	$\epsilon\ \zeta\ \eta\ \theta$	$A = AB$	II.	49
18	II.	$b = bC$	$\epsilon\ \zeta\ \eta$	$b = abC$	VII.	50
19	XII.	$Ab = bC$	$\epsilon\ \zeta\ \theta$	$b = abc$	VII.	51
20	VII.	$b = AbC$	$\epsilon\ \zeta$	$b = 0$		52
21	XII.	$Ac = Bc$	$\epsilon\ \eta\ \theta$	$A = AB,\ aB = aBC$	IX.	53
22	VII.	$c = ABc$	$\epsilon\ \eta$	$A = AB,\ c = Ac$	V.	54
23	XIII.	$a = bc \cdot\!\!\mid\!\!\cdot aBC$	$\epsilon\ \theta$	$b = ac$	VI.	55
24	X.	$a = aBC,\ Ab = AbC$	ϵ	$b = 0$		56
25	XI.	$Ab = AbC,\ aB = aBc$	$\zeta\ \eta\ \theta$	$A = AB,\ aB = aBc$	IX.	57
26	IX.	$b = bC,\ BC = ABC$	$\zeta\ \eta$	$b = aC$	VI.	58
27	IX.	$C = AC,\ Ac = ABc$	$\zeta\ \theta$	$C = AC,\ A = AB$	V.	59
28	V.	$C = AC,\ b = bC$	ζ	$b = 0$		60
29	IX.	$a = ab,\ Ab = AbC$	$\eta\ \theta$	$A = B$	I.	61
30	V.	$a = ab,\ b = bC$	η	$a = b = bC$	IV.	62
31	VI.	$a = bc$	θ	$a = b = bc$	IV.	63
32		$a = 0$		$a = 0,\ b = 0$		64

THE LOGICAL INDEX—*continued*

Number.	Type.	Present a (not β), γ.	Present.	Present a (not β) not (γ).	Type.	Number.
65	VIII.	$AB = ABC$	$\delta\ \epsilon\ \zeta\ \eta\ \theta$	$AB = AC$	XII.	97
66	XII.	$Ac = bc$	$\delta\ \epsilon\ \zeta\ \eta$	$A = bc \cdot\!\!\mid\!\!\cdot ABC$	XIII.	98
67	XI.	$AB = ABC,\ ab = abc$	$\delta\ \epsilon\ \zeta\ \theta$	$b = bc,\ Bc = aBc$	IX.	99
68	IX.	$a = aB,\ AB = ABC$	$\delta\ \epsilon\ \zeta$	$b = Ac$	VI.	100
69	II.	$B = BC$	$\delta\ \epsilon\ \eta\ \theta$	$B = BC,\ bC = abC$	IX.	101
70	VII.	$c = Abc$	$\delta\ \epsilon\ \eta$	$c = Ab$	VI.	102
71	IX.	$B = BC,\ bC = AbC$	$\delta\ \epsilon\ \theta$	$B = C$	I.	103
72	V.	$a = aB,\ B = BC$	$\delta\ \epsilon$	$b = c = Ac$	IV.	104
73	XII.	$AB = BC$	$\delta\ \zeta\ \eta\ \theta$	$A = BC \cdot\!\!\mid\!\!\cdot Abc$	XIII.	105
74	XIII.	$a = Bc \cdot\!\!\mid\!\!\cdot abC$	$\delta\ \zeta\ \eta$	$A = BC \cdot\!\!\mid\!\!\cdot bc$	XIV.	106
75	IX.	$C = AC,\ Ac = Abc$	$\delta\ \zeta\ \theta$	$C = AB$	VI.	107
76	VI.	$a = Bc$	$\delta\ \zeta$	$a = aBc,\ A = ABC \cdot\!\!\mid\!\!\cdot Abc$	XV.	108
77	VII.	$B = ABC$	$\delta\ \eta\ \theta$	$B = AC$	VI.	109
78	X.	$a = abC,\ AB = ABC$	$\delta\ \eta$	$a = abC,\ A = ABC \cdot\!\!\mid\!\!\cdot Abc$	XV.	110
79	V.	$B = BC,\ C = AC$	$\delta\ \theta$	$B = C = AC$	IV.	111
80		$a = 0$	δ	$a = 0$		112
81	II.	$A = AC$	$\epsilon\ \zeta\ \eta\ \theta$	$A = ABC$	VII.	113
82	VII.	$c = aBc$	$\epsilon\ \zeta\ \eta$	$A = ABC,\ ab = abC$	X.	114
83	IX.	$c = ac,\ aC = aBC$	$\epsilon\ \zeta\ \theta$	$b = bc,\ c = ac$	V.	115
84	V.	$c = ac,\ a = aB$	$\epsilon\ \zeta$	$b = 0$		116
85	VII.	$c = abc$	$\epsilon\ \eta\ \theta$	$A = AB,\ B = BC$	V.	117
86		$c = 0$	$\epsilon\ \eta$	$c = 0$		118
87	VI.	$c = ab$	$\epsilon\ \theta$	$b = c = ac$	IV.	119
88		$c = 0$	ϵ	$b = 0,\ c = 0$		120
89	IX.	$c = ac,\ aC = abC$	$\zeta\ \eta\ \theta$	$A = BC$	VI.	121
90	VI.	$c = aB$	$\zeta\ \eta$	$A = ABC,\ a = aBc \cdot\!\!\mid\!\!\cdot abC$	XV.	122
91	I.	$A = C$	$\zeta\ \theta$	$A = C = AB$	IV.	123
92	IV.	$a = c = aB$	ζ	$b = 0$		124
93	V.	$c = ac,\ a = ab$	$\eta\ \theta$	$A = B = BC$	IV.	125
94		$c = 0$	η	$c = 0$		126
95	IV.	$a = c = ab$	θ	$A = B = C$	III.	127
96		$a = 0,\ c = 0$		$a = 0,\ b = 0,\ c = 0$		128

THE LOGICAL INDEX—*continued*

Number.	Type.	Present (not a), β, γ.	Present.	Present (not a) β (not γ).	Type.	Number.
129	VIII.	$AB = ABc$	δ ϵ ζ η θ	$A = Ac$	II.	161
130	XI.	$AB = ABc,\ ab = abC$	δ ϵ ζ η	$C = aC,\ ac = aBc$	IX.	162
131	XII.	$AC = bC$	δ ϵ ζ θ	$C = aBC$	VII.	163
132	IX.	$a = aB,\ AB = ABc$	δ ϵ ζ	$C = aC,\ a = aB$	V.	164
133	XII.	$AB = Bc$	δ ϵ η θ	$C = aC,\ ac = abc$	IX.	165
134	IX.	$c = Ac,\ AC = AbC$	δ ϵ η	$a = C$	I.	166
135	XIII.	$a = BC \cdot\!\|\!\cdot abc$	δ ϵ θ	$C = aB$	VI.	167
136	VI.	$a = BC$	δ ϵ	$a = C = aB$	IV.	168
137	II.	$B = Bc$	δ ζ η θ	$C = abC$	VII.	169
138	IX.	$B = Bc,\ bc = Abc$	δ ζ η	$C = ab$	VI.	170
139	VII.	$C = AbC$	δ ζ θ	$C = 0$		171
140	V.	$a = aB,\ B = Bc$	δ ζ	$C = 0$		172
141	VII.	$B = ABc$	δ η θ	$C = aC,\ a = ab$	V.	173
142	V.	$B = Bc,\ c = Ac$	δ η	$a = C = ab$	IV.	174
143	X.	$a = abc,\ AB = ABc$	δ θ	$C = 0$		175
144		$a = 0$	δ	$a = 0,\ C = 0$		176
145	XII.	$AB = Ac$	ϵ ζ η θ	$A = ABc$	VII.	177
146	IX.	$b = bC,\ BC = aBC$	ϵ ζ η	$b = bC,\ C = aC$	V.	178
147	XIII.	$A = bC \cdot\!\|\!\cdot ABc$	ϵ ζ θ	$A = ABc,\ ab = abc$	X.	179
148	VI.	$b = AC$	ϵ ζ	$b = 0$		180
149	XIII.	$A = Bc \cdot\!\|\!\cdot AbC$	ϵ η θ	$A = Bc$	VI.	181
150	VI.	$c = AB$	ϵ η	$A = c = AB$	IV.	182
151	XIV.	$A = Bc \cdot\!\|\!\cdot bC$	ϵ θ	$A = ABc,\ a = aBc \cdot\!\|\!\cdot abc$	XV.	183
152	XV.	$a = aBC,\ A = ABc \cdot\!\|\!\cdot AbC$	ϵ	$b = 0$		184
153	IX.	$B = Bc,\ bc = abc$	ζ η θ	$A = AB,\ B = Bc$	V.	185
154	I.	$b = C$	ζ η	$b = C = aC$	IV.	186
155	VI.	$C = Ab$	ζ θ	$C = 0$		187
156	IV.	$b = C = AC$	ζ	$b = 0,\ c = 0$		188
157	VI.	$B = Ac$	η θ	$A = B = Bc$	IV.	189
158	IV.	$B = c = Ac$	η	$A = B = c$	III.	190
159	XV.	$a = abc,\ A = AbC \cdot\!\|\!\cdot ABc$	θ	$C = 0$		191
160		$a = 0$		$a = 0,\ b = 0,\ C = 0$		192

THE LOGICAL INDEX—*continued.*

Number.	Type.	Present (not α) (not β) γ.	Present.	(not α) (not β) (not γ).	Type.	Number.
193	II.	A = A*b*	δ ε ζ η θ	A = A*bc*	VII.	225
194	IX.	A = A*b*, *ab* = *ab*C	δ ε ζ η	A = *bc*	VI.	226
195	IX.	A = A*b*, *ab* = *abc*	δ ε ζ θ	A = A*b*, *b* = *bc*	V.	227
196	I.	*a* = B	δ ε ζ	A = *b* = *bc*	IV.	228
197	VII.	B = *a*BC	δ ε η θ	B = BC, C = *a*C	V.	229
198	V.	*c* = A*c*, A = A*b*	δ ε η	A = *c* = A*b*	IV.	230
199	VI.	B = *a*C	δ ε θ	B = C = *a*C	IV.	231
200	IV.	*a* = B = BC	δ ε	A = *b* = *c*	III.	232
201	VII.	B = *a*B*c*	δ ζ η θ	A = A*bc*, *a*B = *a*B*c*	X.	233
202	VI.	B = *ac*	δ ζ η	A = A*bc*, *a* = *ab*C ꖌ *a*B*c*	XV.	234
203	V.	C = AC, A = A*b*	δ ζ θ	C = o		235
204	IV.	*a* = B = B*c*	δ ζ	C = o		236
205		B = o	δ η θ	B = o		237
206		B = o	δ η	B = o		238
207		B = o	δ θ	B = o, C = o		239
208		*a* = o, B = o	δ	*a* = o, B = o, C = o		240
209	VII.	A = A*b*C	ε ζ η θ	A = o		241
210	V.	A = A*b*, *b* = *b*C	ε ζ η	A = o		242
211	VI.	A = *b*C	ε ζ θ	A = o		243
212	IV.	A = *b* = *b*C	ε ζ	A = o, *b* = o		244
213	X.	A = A*b*C, *a*B = *a*BC	ε η θ	A = o		245
214		*c* = o	ε η	A = o, *c* = o		246
215	XV.	A = A*b*C, *a* = *a*BC ꖌ *abc*	ε θ	A = o		247
216		*c* = o	ε	A = o, *b* = o, *c* = o		248
217	V.	B = B*c*, *c* = *ac*	ζ η θ	A = o		249
218	IV.	B = *c* = *ac*	ζ η	A = o		250
219	IV.	A = C = A*b*	ζ θ	A = o, C = o		251
220	III.	A = *b* = C	ζ	A = o, *b* = o, C = o		252
221		B = o	η θ	A = o, B = o		253
222		B = o, *c* = o	η	A = o, B = o, *c* = o		254
223		B = o	θ	A = o, B = o, C = o		255
224		*a* = o, B = o, *c* = o		All absent		256

CHAPTER XXIX

MISCELLANEOUS QUESTIONS AND PROBLEMS

It seems convenient to bring these Studies in Deductive Logic to a close by adding a certain number of mixed Questions and Problems, which may refer to any part of logical doctrine. In some cases these questions pass the bounds of formal and deductive logic. It is left to the student to determine what part, if any, of the preceding pages will assist him. To certain questions, however, are appended references to other works where the proper assistance will be found.

1. What may we expect to happen, in a logical point of view, when an irresistible force meets with an infinite resistance?

2. If it is said to be false that what has the properties A and B has those also of C and D, and *vice versâ*, how would you interpret this statement as affecting the possible relations of A, B, C, and D?

3. In how many ways may we in a purely logical point of view contradict the assertion of Hobbes that 'Irresistible might in the state of nature is right'? (See p. 182, Question 7.) Specify the ways.

4. Analyse the logical import of the following passage from the *Wealth of Nations*, book i. chapter viii. :—

'It is not because one man keeps a coach, while his neighbour walks afoot, that the one is rich, and the other poor; but because the one is rich, he keeps a coach, and because the other is poor, he walks afoot.'

5. In Harriet Martineau's *Autobiography* (vol. i. p. 355) we are told that a certain lady, after receiving from Charles Babbage a long explanation of his celebrated calculating machine, terminated the conversation with the following question: 'Now, Mr. Babbage, there is only one thing more that I want to know. If you put the question in wrong, will the answer come out right?'

If you think this question absurd, give distinct and detailed reasons for thinking so, and reconcile them with the fact that false premises may give a true conclusion.

6. Explain and illustrate the Aristotelian saying: *Ex veris fieri non potest ut falsum concludatur; ex falsis contra verum;* and show some of its applications in the investigation of nature. [R.]

7. A certain argument having been shown to involve paralogism, inquire into the conditions under which this failure does or does not tend to establish the contradictory conclusion.

8. In a certain borough, on one occasion, the Liberal party objected to 3624 voters, and the Conservative party to 5531 voters, the whole constituency being 10,000. What is the least number of voters which can have been objected to on both sides? What is the greatest number? What is the most probable number, supposing the objections to be made quite at haphazard?

9. What is the logical, compared with the popular, interpretation of the injunction, 'This man is not on any account to be ducked in the horse-pond.' Explain the difference.

10. What is the logical, compared with the popular, interpretation of the injunction, 'All persons are requested not to discharge fireworks among the crowd around the bonfire on the 5th November.'

11. 'Because a horse is an animal, the head of a horse is the head of an animal.'

Examine the validity of this inference. Can you express the reasoning syllogistically, or symbolically? [E.]

12. Investigate the nature of the reasoning, good or bad, involved in the four following examples :—

(1) Elephants are stronger than horses; horses are stronger than men; therefore elephants are stronger than men. [E.]

(2) Alexander was the son of Philip; therefore Philip was the father of Alexander.

(3) As good kill a man as kill a good book; for he that kills a man does but kill a reasonable creature; but he that kills a good book kills Reason herself.

(4) Nay, look you, I know 'tis true; for his father built a chimney in my father's house, and the bricks are alive at this day to testify it. [O.]

13. What methods underlie these inferences?

Because it froze last night, therefore the pools are covered with ice.

During the retreat of the Ten Thousand a cutting north-wind blew in the faces of the soldiers; sacrifices were offered to Boreas, and the severity of the wind immediately ceased, which seemed a proof of the god's causation. [P.]

14. What method is employed in the following?—

'Brewster accidentally took an impression from a piece of mother-of-pearl in a cement of resin and bees'-wax, and finding the colours repeated upon the surface of the wax, proceeded to take other impressions in balsam, fusible metal, lead, gum-arabic, etc., and always found the iridescent colours the same. He thus proved that the chemical nature is wholly a matter of indifference, and the form of the surface is the condition of such colours.'

15. What is the difference, logically, between the sentences:—Leibnitz, a great philosopher, has said, etc.; and A great philosopher, Leibnitz, has said, etc.? [C.]

16. What is successive induction, or induction by connection, as in the proof that $n^2 = 1 + 3 + 5 + \ldots\ldots$ up to n of the odd numbers? [H.]

(See *Elementary Lessons in Logic*, lesson xxvi. p. 220.)

17. Give an inductive proof that $x^n - a^n$ is divisible by $x - a$, when n is a whole number.

18. Can there be such a thing as a fallacy of simple inspection, that is a fallacy which does not involve inference? (See Mill's *System of Logic*, book iv. chapter iii.)

19. Leibnitz says 'Knowledge is either obscure or clear. The clear is again either confused or distinct: and the distinct either adequate or inadequate; is further either symbolical or intuitive; and if it be at the same time both adequate and intuitive, it is perfect.'

Give an exhaustive classification of all possible kinds of knowledge under the above conditions. (See *Elementary Lessons in Logic*, lesson vii.)

20. Explain the logical meanings of the terms *Genus*, *Species*, *Difference*, *Property*, and *Accident*, distinguishing the meaning in extent and intent, and using for illustration the varieties of things ABCD, ABC*d*, AB*c*D, AB*cd*, in which A, B, C, D are terms denoting qualities, and *c*, *d* the negatives of C, D.

21. Draw the inferences deducible from these data:

(1) A is the only antecedent always present when p is present, and always absent when p is absent.

(2) A is an antecedent always present when p is present, and always absent when p is absent.

(3) A is an antecedent frequently present when p is present, and frequently absent when p is absent. [P.]

22. Point out the exact nature of the relations between the logical processes of Abstraction, Analysis, Synthesis, and Generalisation.

23. What is the logical difference, if any, between nouns Substantive and nouns Adjective?

24. Is a Latin adjective used alone in the neuter an adjective or a substantive?

25. Is there self-contradiction in the assertion that knowledge of what is outside my consciousness may be inside my consciousness?

26. Can absolute certainty be found in any conclusion (1) inductively established, (2) deductively established? [E.]

27. Is there any distinction, and if so what, between a general and an abstract notion, and is there a corresponding difference between the names employed to express them? [E.]

28. It is a rule of syllogism that nothing can be inferred from particular premises. How then can I infer from the particular facts that some men have died, the universal conclusion, 'All men die'? [E.]

29. Explain the limits of demonstrative science, and examine the following statement: 'No matter of fact can be matter of demonstration.' [E.]

30. Distinguish Logical, Mathematical, and Physical Quantity. [E.]

31. Distinguish and exemplify Logical, Mathematical, Metaphysical, Physical, and Moral Necessity. [E.]

32. When two phenomena are causally connected together, can you always ascertain which is the cause and which is the effect? If so, how? [L.]

33. Investigate how far or on what grounds our knowledge of the following propositions approximates to certainty :—

'Nitric acid does not dissolve gold.'

'A distant fixed star is subject to gravity.'

34. Consider from a logical point of view the assertion that the increasing trade of Great Britain is caused by a reform of the tariff. What kind of proof is applicable?

35. A man having been shot through the heart immediately falls dead. Investigate the logical value of such a fact as proving that all men shot through the heart will fall dead. [I.]

36. What do you understand by a 'working hypothesis'? Under what conditions is it legitimate for an investigator to employ hypothesis? (Huxley. Mill's *System of Logic*, book iii. chap. xiv.; *Principles of Science*, chap. xxiii.) [L.]

37. State these arguments formally, and give their technical designations :—

(1) 'The thinking power does not belong to matter; otherwise matter generally would exhibit it.'

(2) 'Happiness is the reward of goodness; and since all do not desire a good life, all cannot obtain its reward.' [P.]

38. Why is it that with exactly the same amount of evidence, both negative and positive, we did not reject the assertion that there are black swans, while we should refuse credence to any testimony which asserted that there were men wearing their heads underneath their shoulders? [P.]

39. What is the difference of meaning, if any, between the propositions, 'This house was built by Jack,' and 'This is the house that Jack built'? (De Morgan, *Third Memoir on the Syllogism*, 10th page.)

40. Does the thesis that the ultimate premises in human knowledge are the result of mental association affect the nature and certainty of Logic, and if so, how? [E.]

41. Define evidence. Distinguish intuitive, demonstrative, and probable evidence. [E.]

42. Explain :—'Certainty, therefore, has for its opposite, *uncertainty* in one way—*impossibility* in another. Uncertainty, in the language of logicians, is its contradictory opposite—impossibility, its contrary opposite.' [P.]

43. Investigate the question whether the truth of a statement is to be judged by the impression which it makes upon those to whom it is addressed, by its literal correspondence with the belief of the person making it, or by any other standard. [L.]

44. It has been pointed out by Ohm that reasoning to the following effect occurs in some works on mathematics: 'A magnitude required for the solution of a problem must satisfy a particular equation, and as the magnitude x satisfies this equation, it is therefore the magnitude required.' Examine the logical validity of this argument.

45. It is probable that Herodotus recorded only what he heard concerning Ethiopia; and it is not unlikely that most that he heard was correct; so that we may accept his account as true. Is this conclusion correct?

46. There is a very strong probability that the eldest child of a newly married couple will inherit the estate of the husband. For, firstly, it is more probable than not that there will be children of the marriage. Next, if a child is born, it is more probable that it will be a son, for more boys

are born than girls. Thirdly, if a son be born, it will probably survive its father. Examine this inference. [O.]

47. Consider the following argument:—'Many writings that are not genuine were ascribed to Clemens Romanus; this Epistle was ascribed to him; therefore this Epistle is not genuine.' [L.]

48. A student of geometry examines three isosceles triangles and finds them agree in having equal angles at the base; an excise officer examines three bottles of wine out of a quantity imported and finds them agree in strength; a chemist analyses three specimens of a mineral and finds them agree in composition: compare the inferences which may be drawn in these cases.

49. What is the relation between classification and induction in general? [L.]

50. When an experiment designed to produce a phenomenon fails to produce it, in how many ways may we interpret or explain the meaning of the failure? [L.]

51. In what ways may a physicist hope to explain away an exceptional phenomenon? [L.]

52. If we never find skins except as the teguments of animals, we may safely conclude that animals cannot exist without skins. If colour cannot exist by itself (*ἅπαν γὰρ χρῶμα ἐν σώματι*), it follows that neither can anything that is coloured exist without colour. So, if language without thought is unreal, thought without language must also be so. What do you think of this argument? [O.]

53. If we are disposed to credit all that is told us, we must believe in the existence not only of one, but of two or three Napoleon Buonapartes; if we admit nothing but what is well authenticated, we shall be compelled to doubt the existence of any. How, then, can we be called upon to believe in the one Napoleon Buonaparte of history? [O.]

54. Brown asserts that all planets are spheroids; Jones denies it; Robinson asserts that Jones knows nothing about the matter; Smith proves that in this case at least Robinson is correct; but Thomson refuses to accept the premises of Smith's proof. What are the logical relations of the parties?

55. From the statement that blood-vessels are either veins or arteries, does it follow logically that a blood-vessel, if it be a vein, is not an artery? Give your reasons.

56. It is asserted by some philosophers that all knowledge is inductive in its origin, and it is generally allowed that inductive inferences can be probable only; if so, no knowledge can be more than probably true. Can you, however, adduce any instance of knowledge which is certainly true? In that case explain the difficulty which evidently arises.

57. *Aut amat aut odit mulier; nihil tertium.* If any one takes upon himself simply to deny the truth of this saying of Publius Syrus, in how many different ways may the denial be interpreted?

58. Explain the following apparent paradox:—P thinks of an object; Q is absolutely ignorant of the size of that object; to him, therefore, the probability that the object is greater than a cannon-ball is $\frac{1}{2}$. Again, being absolutely ignorant about its size, he has no reason to believe it either greater or less than a pea, the probability of either case being $\frac{1}{2}$. Hence to him it is infinitely improbable that the object is intermediate in size between a pea and a cannon-ball. [JOHN HOPKINSON, D.SC.]

59. In defending a prisoner his counsel must either deny that the deed committed is a crime, or he must deny that the prisoner committed the deed; therefore if the counsel denies that the deed committed is a crime, he must admit that the prisoner did commit the deed.

60. What do you understand by the logical *proof* of an assertion? Compare the logical meaning of the word *proof* with any other meanings of the word known to you. [I.]

61. Can all kinds of propositions be exhibited in the intensive as well as the extensive form? Give reasons in support of your answer; and in the event of its being in the negative, draw up a list distinguishing between those kinds of propositions which can, and those which cannot, be so exhibited. [L.]

62. Explain the meaning of the assertion that Induction is the inverse process of Deduction.

63. Illustrate Mathematical Induction in its several kinds or cases, and discuss its relation to induction in the physical sciences.

64. What is the relation, if any, between the inductive syllogism and the inductive methods employed in the physical sciences?

65. Estimate upon logical grounds the possibility of establishing a school in which students should be rendered capable of discovering the Laws of Nature. (Gore's *Art of Scientific Discovery.*)

66. What precisely is meant by the Law of Continuity? Point out the grounds and limits of its validity. (*Life of Sir W. Hamilton*, p. 439; *Principles of Science*, chapter xxvii.)

67. When the effects of three distinct causes are added and mingled together, by what processes of experiment and reasoning can we assign to each cause its separate effect? [C.]

68. Under what circumstances are we to accept the failure of an experiment or series of experiments as proving the non-existence of the phenomenon intended to be produced? (*Principles of Science*, chapter xix.) [L.]

69. Illustrate the scientific value of exceptional phenomena, and show in how many ways they may be disposed of or reconciled with physical law. (*Principles of Science*, chapter xxix.) [L.]

70. What is the difference between the *causal* and the *casual* happening of events, if, as is generally allowed, not even a dead leaf falls to the ground without sufficient causes to determine the precise moment of its falling and the precise spot upon which it will fall?

71. Show by example that the logical copula does not imply the notion of existence. [E.]

72. Investigate the question whether the functions of affirmative and negative propositions in reasoning are similar.

73. England is the richest country in the world, and has a gold currency. Russia and India, in proportion to population, are poor countries, and have little or no gold currency. How far are such kinds of facts logically sufficient to prove that a gold currency is the cause of a nation's wealth? [I.]

74. If by two distinct methods of investigation you arrive at the same conclusion, namely, that the currency of the kingdom does not exceed one hundred millions sterling, but it is afterwards discovered that one of the methods of investigation involved fallacious reasoning, what would you be inclined to infer about the other method of investigation? [I.]

75. A certain argument having been shown to involve paralogism, inquire into the conditions under which this failure does or does not tend to establish the contradictory conclusion.

76. Investigate the logical, psychological, and moral grounds of the saying, '*Qui s'excuse, s'accuse.*'

77. Taking the senses in which they most resemble one another, distinguish between judgment, opinion, statement, knowledge, fancy, conjecture, supposition, allegation. [E.]

78. Distinguish: truth, certainty, fact, opinion, probability, evidence, conviction. [E.]

79. How far can the inconceivability of the opposite be regarded as proof of the truth of any judgment? [E.]

80. Right-angled and not-right-angled are contradictory predicates; therefore, according to the law of Excluded Middle, as the proposition All triangles are right-angled is false, it must be true that all triangles are not right-angled. But this also is false. Explain the above difficulty. [E.]

81. Given that (1) whenever the statements a, b, x are either all three true, or all three false, then the statement c is false, and y is true, or else c is true, and y is false; (2) that whenever d, e, y are either all three true or all three false, then the statement a is false, and x is true, or a is true, and x is false. When can we infer from these premises that either x or y is true?

[Hugh MacColl, B.A., in *Educational Times*, question 6206. A solution was given by C. J. Monro, M.A., in the same paper for March 1880. The question seems to mean—What other conditions with those given determine that either x or y is true?]

82. De Morgan says (*Fourth Memoir on the Syllogism*, p. 5) of the Laws of Thought: 'Every transgression of these laws is an invalid inference; every valid inference is not a transgression of these laws. But I cannot admit that everything which is not a transgression of these laws is a valid inference.' Investigate the logical relations between these three assertions.

83. To what type of assertion do the premises of Darapti belong?

84. Give the converse, inverse, contrapositive, obverse, and reciprocal propositions of the following:—

(1) All parallelograms have their opposite angles equal.

(2) If P is greater than Q, then R will be greater than S.

(3) Two triangles are congruent if the three sides of the one are respectively equal to the three sides of the other.

85. Why have some mathematicians been accustomed to say that it is necessary to prove the *converse* of a mathematical proposition?

86. Where exactly lies the error of the Irishman, who being charged with theft on the evidence of three witnesses who had seen him stealing the article in question, proposed to bring in his defence thirty witnesses who had not seen him stealing it?

87. Epimenides says that every statement of a Cretan is a lie; but Epimenides is a Cretan; therefore what he says is a lie; therefore every statement of a Cretan is not a lie. [E.]

88. If, in saying that 'few strikes are beneficial,' I feel sure that the statement will be misinterpreted by those to whom it is addressed, and that the statement 'no strikes are beneficial,' although not in my opinion literally true, will more exactly convey to the hearers' minds the impression which I believe to be true, ought I, having regard to the moral obligation of speaking the truth, to use the latter assertion or the former?

89. 'I will go on,' said King James, 'I have been only too indulgent. Indulgence ruined my father.'

Express clearly the process of reasoning involved in this utterance. Is it Induction? or what? [M.]

90. What is the relation, if any, between the inductive syllogism and the inductive methods employed in the physical sciences?

91. Can the proposition, 'All *A* is all *B*,' be regarded as representing a single act of thought? (See *Mind*, vol. i. p. 216.) [E.]

92. Are the premises of Darapti given only in a numerical form sufficient to prove the conclusion?

93. Does it follow that, because some poetry is not in verse, there must be some verse which is not poetry? [H.]

94. Take the proposition, 'All sciences are useful,' and determine precisely what it affirms, what it denies, and what it leaves doubtful, concerning the relations of the terms 'science' and 'useful thing.'

95. Ascertain precisely how many distinct assertions there are in the description of the conduct of the great scholastic logician, John of Salisbury, after Thomas à Becket had been murdered by his side:—*Tacitus sed moerens, continuo se subduxit.*

96. Can you represent equationally the contradiction between 'Some *X*s are not some *Y*s' and 'There is one *X* only and that is the only *Y*'?

97. Which of the types of assertion involving three terms are *complete*, in the sense of admitting no additional assertion involving the same three terms without self-contradiction?

98. If all things are either *X* or *Y*, and all things are either *Y* or *Z*, what inference can you draw?

99. Do the thirty-six moods of Hamilton's Syllogism with quantified predicates (see table in *Elementary Lessons*, p. 188; Thomson's *Laws of Thought*, section 103) comprise all the possible weakened moods?

100. Is the student of logic, generally speaking, prepared *rapidly* to analyse the two following propositions, and to say whether or no they must be identical, if the identity of synonyms be granted?—

(1) The suspicion of a nation is easily excited, as well against its more civilised as against its more warlike neighbours, and such suspicion is with difficulty removed.

(2) When we see a nation either backward to suspect its neighbour, or apt to be satisfied by explanations, we may rely upon it that the neighbour is neither the more civilised nor the more warlike of the two.'

[DE MORGAN, *Third Memoir*, 1858, p. 181.]

101. Is the following proposition a definition or not? Is it on the matter or the form of the proposition that you found your answer?

'LOGICA EST ARS ARTIUM ET SCIENTIA SCIENTIARUM.'

THE END

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